Fractal Analysis of Laurentide Glacial Boundaries in the Conterminous United States
An Honors Thesis (HONRS 499)

by

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Date: May 2005

Expected Date of Graduation
May 7, 2005

A brief explanation

The purpose of this Honors Thesis was to learn how interpret fractal data and write a scientific paper. I have based this paper on the data I obtained through my Undergraduate Research Fellowship. During this fellowship I worked with Dr. Scott Rice-Snow in developing a project and carrying out the actual experiment.

Writing is an important skill that is often emphasized in academia. Students are taught how to create proper sentence structure, use concise and colorful verbs and adjectives and form smooth transitions. Some form of creative writing or opinion paper is often the medium of choice for learning such skills. Writing is also very important in the world of science. However, scientific writing is different from that of creative writing. Scientific writing is designed to be clear, simple and to the point. It does no need to include creative adjectives and exciting verb choice. To be quite honest, it should be bland. This structure is used to inform the audience about an important issue and to document the particular approach they used to investigate that issue.

In my studies of geology over the past four years I have read a large number of scientific papers. I have taken away much knowledge of the form of scientific writing over this time. The following is a brief synopsis of what I have learned.

A scientific paper is composed of many sections. Required sections vary from publication to publication. There are a few sections that are considered standard such as: Abstract, Introduction, Methods (and Materials), Results, Discussion and References. Generally, a Conclusions section is included as well. There are also a few guidelines to each of these sections.

The Abstract is a very important portion of the paper. In fact, this is often the only part of the paper that is read by those who are searching for pertinent information. The Abstract should contain the purpose of the study, the methods used, the results and the importance of these results. This section should be written in past tense.

The Introduction is a vital part of the article. This portion of the paper is dedicated to the background information that will help explain the current study, as well as the question and/or hypothesis addressed with the study. Present tense should be used when discussing the background material, while past tense should be used when referring to the experiment.

The Methods (and Materials) section is used to explain the experiment in a manner in which those who wish to can repeat the process. This should not be a simple list of steps; it should be written in paragraph form and in past tense.

The Results section should include things such as tables or charts. There should be a brief description of the data but all interpretations and conclusions should be left out of this portion of the paper.

The Discussion provides the appropriate platform to analyze and expand on the results. This is also a good section of the paper for comparing your results to that of previous studies. Often the Discussion is where the author should address the connection of the results with the objective of the study.

Many times books about scientific writing offer ideas for a proper Reference section. However, I have learned that it is best to obtain a list of regulations from the journal in which one is intending to publish with to find the format in which they would like to have the references displayed. For my References section I have chosen to use the form recommended by *Geomorphology* (a scientific journal) which is a likely journal to publish a work such as my paper.

I have opted to include a separate section entitled Conclusions in my paper. I found that this section is helpful for readers to get a better feel for the results and interpretations of the study in a quick fashion. This section is a summary of the article as well as a place to put final conclusions and statements. I have also created an Acknowledgments section which I have noted in many papers and feel in my case it is necessary to say thanks to those who made the research this paper was based on possible.

**Note: The abstract to this thesis can be found on Page 1 of the scientific paper. The Acknowledgments section (Page32) will not include any thanks for Dr. Rice-Snow because as the paper has been written, he is a co-author and therefore it would not be appropriate to mention him in the Acknowledgments section. However, I would like use this space to thank Dr. Rice-Snow for being the inspiration of this thesis, the guidance behind my success and for the editing of this paper.

Fractal Analysis of Laurentide Glacial Boundaries in the Conterminous United States

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Abstract

The Wisconsinan event of the Laurentide Ice Sheet covered a wide fetch of land from New Jersey to Montana in the conterminous United States. The contact it created between new tills and older units formed a boundary that exhibits irregular curves with many scales of wandering. This boundary crosses five physiographic provinces: the Piedmont, Valley and Ridge, Appalachian Plateau, Central Lowlands and Great Plains Provinces. We have performed divider analyses on the Wisconsinan maximum advance to correlate plan-view geometry of the boundary with local geomorphology. In addition to dividing the boundary into sections according to province, large provinces were also split up into smaller segments to reduce the issue of scale discrepancy. A total of 37 line segments were created including: five of the whole province sections and 32 subsequent divisions.

All sections of this boundary of maximum advance were found to have fractal character. Richardson plots showed at least one area of linearity and in some cases two. These linear areas led to calculated fractal dimension (D) values. The ranges of these D values are quite broad. The fine scales showed values of 1.01-1.17 and the coarse scales had values of 1.01-1.32. The Appalachian Plateau Province shows a fractal character pattern that is different from that of most other plots, where the amount of wandering decreases in the coarse scales. The subsections of this area also show D values that occur over similar ranges of step lengths. These studies are significant, as they may imply an active scale-specific process in the formation of these boundary segments.

Introduction

The Laurentide Ice Sheet of the Late Pleistocene left enormous scars and deposits on the North America Continent. During this period of advance the ice sheet descended from Canada into the United States as far south as portions of Kentucky. These glacial deposits were left predominantly in the form of ground and terminal moraines. The glacial till units overlie previously deposited units of bed rock and older glacial tills. The present day boundaries of maximum advance are designated by the accumulation of debris in front of the glacier, terminal moraines, or the smearing out and deposition of debris underneath the glacier, ground moraines. The contacts between these stratigraphic units and those of previous deposition form irregular curves.

A fractal is a geometrical shape that can be subdivided into fragments that are exact or approximately reduced versions of the original form. This quality is called self-similarity. When a figure or line is too complex to define using a simple equation, a fractal property such as a fractal dimension can be utilized as a descriptive property.

The goal of this research is to determine what geologic factors influenced the formation of the Laurentide Ice Sheet boundary and in what ways these factors influenced this boundary. Fractal dimensions of specific areas will be calculated and matched to a geologic process so that comparisons can be made between areas of similar dimension and/or geological setting. This will reinforce the hypothesis that there is a connection between dimension values and geology or it will allow for exploration of other hypotheses.

The boundaries in this study are included in the conterminous United States, and extend from the Atlantic shores of New Jersey to the Canadian border in northwestern

Montana. Having been exposed to erosive forces and being unsupported or unsupplied by glacial actions, the boundary of maximum advance has undergone some deterioration. This deterioration also leads to difficulty in human interpretation and mapping of boundary locations. Some amount of non-systematic error is then added into this study. It should also be noted that this boundary cannot be mapped with complete precision. Regardless of the scales of the maps used, small scale irregularities and curves are neglected and therefore smoothing will occur. This smoothing will also lead to a component of systematic error that will result in the underestimation of the length on the boundary.

It has been suggested for some time that a change in the nature of local geomorphological processes leads to a change in the form of land surfaces (Elliot, 1989). These landforms are often of complicated shape and form and are therefore a challenge to analyze by conventional means. Thusly a fractal analysis is employed. The divider method is commonly used for evaluation of fractal character in natural curves (Snow and Mayer, 1992).

Divider analysis provides a summary of the geometry (wiggliness or wandering) of an irregular curve in two dimensions. In this method a pair of dividers is used to measure the length of an irregular line by breaking up the line into straight segments called steps. The length of the line is repeatedly walked off by a real or virtual divider at different steps sizes. The length of the line is then calculated by multiplying the number of steps by the step size. This data of length versus step size is then plotted on a doubly-logarithmic graph where fractal dimension be read as a function of the slope of the line that is formed (Mandlebrot, 1983).

Fractal analyses have been used to study many different geological landforms including: coast line form (Mandelbrot, 1967), mountain-fronts (McClelland, 1985) and river channel patterns (Snow, 1989; Nikora, 1991). Significant work has also been done on drainage basin boundaries. Snow (1998) used fractal analyses to compare the geometries of basins to local rock types and climates. Glacial features have also been previously examined using this technique. These features include: glacial moraines (Elliot, 1989), glacial surface structures (Bishop, 1998; Szekely, 1999) and glacial grooves (Snow, Lowell and Rupp, 1991).

In the study of the glacial grooves, researchers showed that there was a change in character from small scale grooves to larger scale grooves. This could mean that there were two separate processes that created these features or that the same process yields different values at varying scales (Snow, Lowell and Rupp, 1991).

For this study we have performed a divider analyses on the Laurentide Ice Sheet glacial maximum boundary. The purpose of the analyses was to observe a correlation in local geomorphologies with fractal dimensions (D). The length of the boundary was split up according to physiographical provinces. Three provinces are in some cases orders of magnitude larger than the remaining two. Therefore these three provinces have been further divided into smaller sections to allow for comparison with the smaller provinces without the need to consider the error involved with a large difference in scales. Comparisons were then made between areas of similar geomorphologies and their corresponding D values.

Materials and Methods

The boundary of maximum advance of the Wisconsinan Laurentide Ice Sheet was created by compiling segments from fourteen maps (for complete listing of these maps see Appendix 1). These maps were 1°x2° Quadrangle maps, state coverage maps and 4°x6° Quadrangle maps of 1:250 000, 1:500 000, and 1:1 000 000 scales respectively. The fourteen maps provided coverage from the Canadian border in western Montana to the Atlantic shore of New Jersey and thus completed the boundary within the conterminous United States.

The maps were digitized using computer software and a digitizing table. The line segments from separate maps were then assembled in a computer aided drafting (CAD) program to form one continuous line spanning the entire distance of the glacial boundary.

The line was then split up according to physiographic provinces (Figure 1). The points where the glacial boundary intersected the boundaries of the provinces were chosen as the points of division. The divisions were made in the same CAD program used to assemble the map segments. These provinces include the Piedmont, Valley and Ridge, Appalachian Plateau, Central Lowlands and the Great Plains (Fenneman and Johnson, 1946).

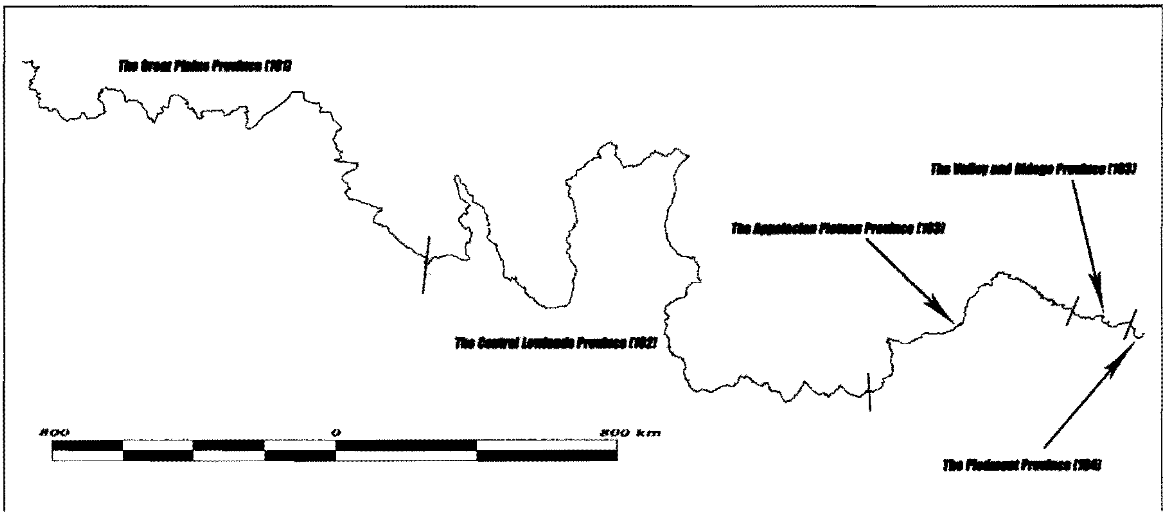


Figure 1. The Laurentide Ice Sheet boundary of maximum advance showing the points of intersection with the physiographic provinces.

A brief description of the provinces

The Piedmont Province

This area is defined by an elongate plateau of moderate relief (150-300m) that extends from New York to Alabama. This ridge forms the eastern boundary of the Appalachian Mountain chain. The region consists of folded and faulted metamorphic and igneous rocks of Paleozoic age. Once controlled by the large-scale unwrapping created by the collision of continental plates, the Piedmont province is now dominated by erosion.

The Valley and Ridge Province

This region is composed of a series of alternating valleys and ridges that run parallel to one another in a general northeast/southwest fashion. The Valley and Ridge Province extend from New York to Alabama. The ridges were created when Paleozoic sedimentary rocks were thrust and folded as the continental collision occurred that ultimately formed Pangaea. The linear arrangement of this province has created a systematic fluvial pattern that is oriented parallel and perpendicular to the ridges.

The Appalachian Plateau Province

During Pennsylvanian and Mississippian time the massive Appalachian Mountains were eroding. Streams carried debris down the mountains and deposited it in huge alluvial fans. Sediments collected on top of the gently folded Paleozoic rocks to the west of the Ancient Appalachians. Today this is a flat area that is dominated by a dissected fluvial landscape of more random orientation.

The Central Lowlands Province

This is the largest of the provinces. It extends from Minnesota to Texas and from Ohio to Kansas. The portion of this area included in this study is characterized by large stretches of land that have been flattened by previous glacial advances. There exists a thick deposit of glacial till on top of bed rock in these areas. This loose deposit is very easily eroded by water and wind action. Thicknesses of the tills increase to the north.

The Great Plains Province

The Great Plains span from western North Dakota and eastern Montana southward to west Texas and eastern New Mexico. The flat surface slopes eastward from the foot of the Rocky Mountains, creating an easterly flow of rivers and streams. The bedrock was created in much the same fashion as the Appalachian Plateau area as the Rocky Mountains were eroded.

The resulting five new line segments, representing the five physiographic provinces, were then analyzed using the divider method. The divider analysis is performed by a computer program. The smallest step was set to be no smaller than the average point spacing and the largest step size was defaulted to the distance from one end of the line to the other. The divider was walked along the line 200 times with 200 different step sizes. This measurement was done from 50 random starting points on each line segment. The output data of the program was then input into a graphing program that was used to construct Richardson plots. Doubly logarithmic plots were produced comparing the step length to the trace length of the line. The graphs were evaluated to find fractal character by obtaining slopes of linear segments within the plotted figure. The fractal dimensions (D) were calculated to be one plus the absolute value of the slope of the generated line.

$$D=1+|m|$$

The same five segments were then broken into similar size portions to allow for further analysis at similar scales of resolution. The segments with significantly larger length (i.e. the Great Plains, Central Lowlands and Appalachian Plateau provinces) were split up into 200-300 km sections. The length of 200-300 km was chosen to make these subdivisions approximately the same length of as the Valley and Ridge Province and thus remove the error of the difference in section size. There were no special considerations for the placement of divisions of the larger provinces into smaller sections besides length of the subsequent segment.

The remaining provinces, the Valley and Ridge and Piedmont Provinces, were left at the original length due to the shortness of the original section. The Great Plains

Province was separated into eleven sections; the Central Lowlands Province was separated into seventeen sections and the Appalachian Plateau into four new sections. With the addition of the two original sections from the smaller provinces, 34 segments of similar length were created. The same process was carried out as before and all new segments were analyzed for fractal character.

The methodology of choosing linear segments within a Richardson plot is not an automated process that is carried out by a computer program. Once the plots were created in a graphing program, the figure was viewed to find straight line segments. When a linear segment was suspected a trend line was put through the data points in this area of the plot. It could then be observed whether this set of points had a true linear tendency. If the line did not fit the data well it was removed or modified to find a better representation of the data.

There were issues to consider when plotting trend lines and locating areas of fractal character. In the finer scale region, the actual limit of detail deserves significant deliberation when translating the data present. A large majority of the Laurentide Ice Sheet boundary was digitized from maps that were of 1:1 000 000 scales (1 mm = 1km). At such scales there is a certain amount of boundary smoothing that occurs and translates to less accuracy of the actual geometry of the line. The limitation of resolution of these maps is the in the range of 3-5 km. Therefore, the sections of the Richardson plots that relate to step sizes finer than this limit will not be evaluated for fractal characteristics. In fact, these sections of the plots are typically curves asymptotic to zero slope, an indicator that the curve is smooth at those fine resolutions.

There also lies a limitation to fractal character determination in the coarser scales. At very large step lengths there issues involving the small sample size and error due to partial step length calculations. In general trends seen within half and order of magnitude from the end of the plot are not included. For a fractal dimension to be considered significant in these areas of the plot, the linear trend must continue into finer scales to prove the same character lies outside of this questionable region.

The scale range describes at what step sizes a fractal character is exhibited. At fine scales the D value can be found when the divider is using a small step size to walk off a line (approximately 3-10 km). When a D value is found in the coarser scales fractal character is revealed as the divider is walking off the length of the line at larger step lengths (approximately 10-1000 km). These scales and ranges were derived by inspection of the data and characteristics of map resolution. If a D value spans across a wide scale range, the line exhibits fractal character at a wide range of step lengths and at a short scale range, the fractal character will be limited to a small range of step lengths.

Results

The results of this study are organized by physiographic province. A description of the data from each province as well as any subsequent divisions will be presented separately. Comparisons will be presented in the discussion.

Thirty-seven Richardson plots have been produced through the process of digitization, melding and splitting of sections, virtual divider calculations and graphing. These 37 graphs include the original divisions of the Laurentide Ice Sheet maximum advance boundary into the 5 provinces. These divisions were numbered 101-105 from west to east (Figure 1). The additional 32 were created by further division of the larger province segments. Divisions of the Great Plains Province were numbered 201-211 from west to east. Central Lowlands sections are labeled 301-317; and Appalachian Plateau 401-404 likewise. All Richardson plots can be seen in Appendix 2. Statistical analysis of fractal data, including the D values of all sections, can be seen in Appendix 3 A-D.

The Great Plains Province

The Richardson plot of the Great Plains Province (Figure 2) shows one linear segment that implies fractal characteristics. The D value that we obtain from the slope of this line is 1.21. This D value represents the fractal dimension of the Laurentide Ice Sheet within the Great Plains Province at fine scales coarse scales.

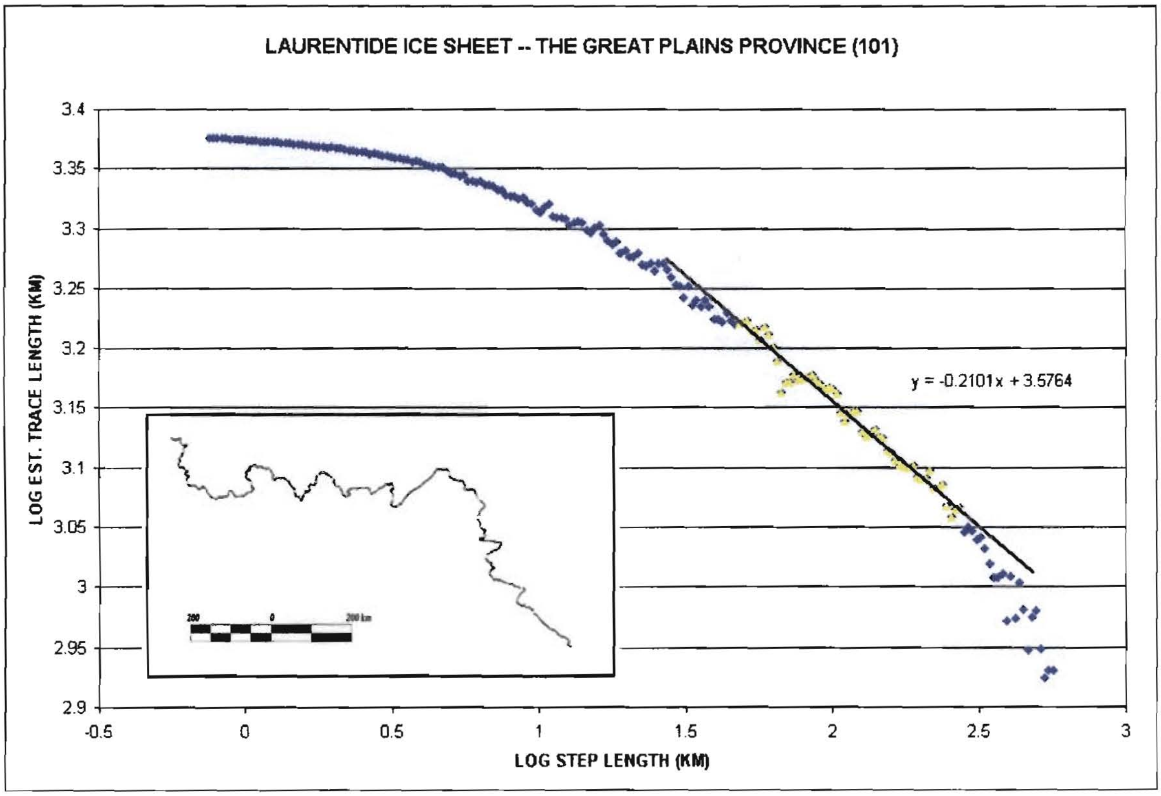


Figure 2. Richardson plot of the Laurentide Ice Sheet glacial maximum within the Great Plains Province.

The subdivisions of the Great Plains Province can be seen in Figure 3. Eleven Richardson plots from the Great Plains Province display a plot trend of increasing slope from finer scales to coarser scales. Some plots such as 201 show D values that extend from the fine scales to the coarse scales. This is means that the line exhibits the same fractal character in both fields. For statistical purposes this scenario will be the same as if the line had two separate linear trends with the same D. Four of the 11 segments from the Great Plains Province show similar situations (201, 208, 209, 211). Five segments have D values only in the coarse scales (202, 204, 205, 206, 210). Only one segment shows a single linear segment in the fine scales (203). Segment 207 show two distinct fractal dimensions. The range of the fine scale D values is quite large (1.01-1.17); as is the range at coarse scales (1.05-1.24). There does not appear to be any systematic changes in D value or step length range within the segments from west to east.

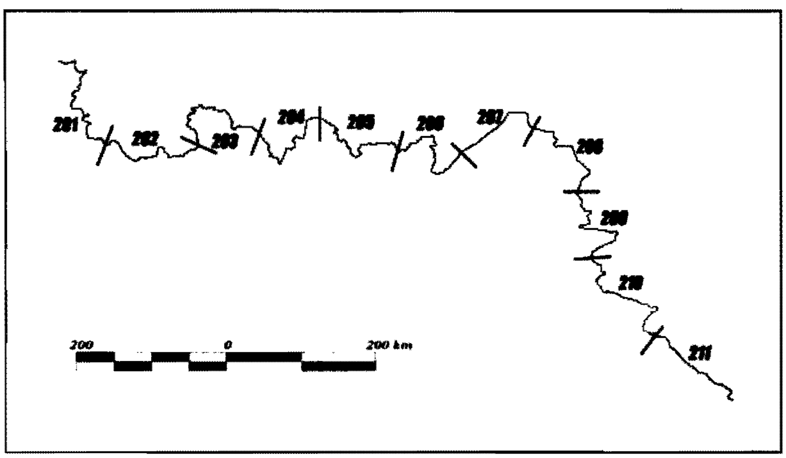


Figure 3. Subdivisions of the Great Plains Province.

The Central Lowlands Province

The Richardson plot of the Central Lowlands Province (Figure 4) looks similar to that of the Great Plains (Figure 2). However, this graph produces two D values of 1.07 and 1.32 in the fine and coarse scales respectively. A trend exists showing an increase in fractal dimension as the step length increases.

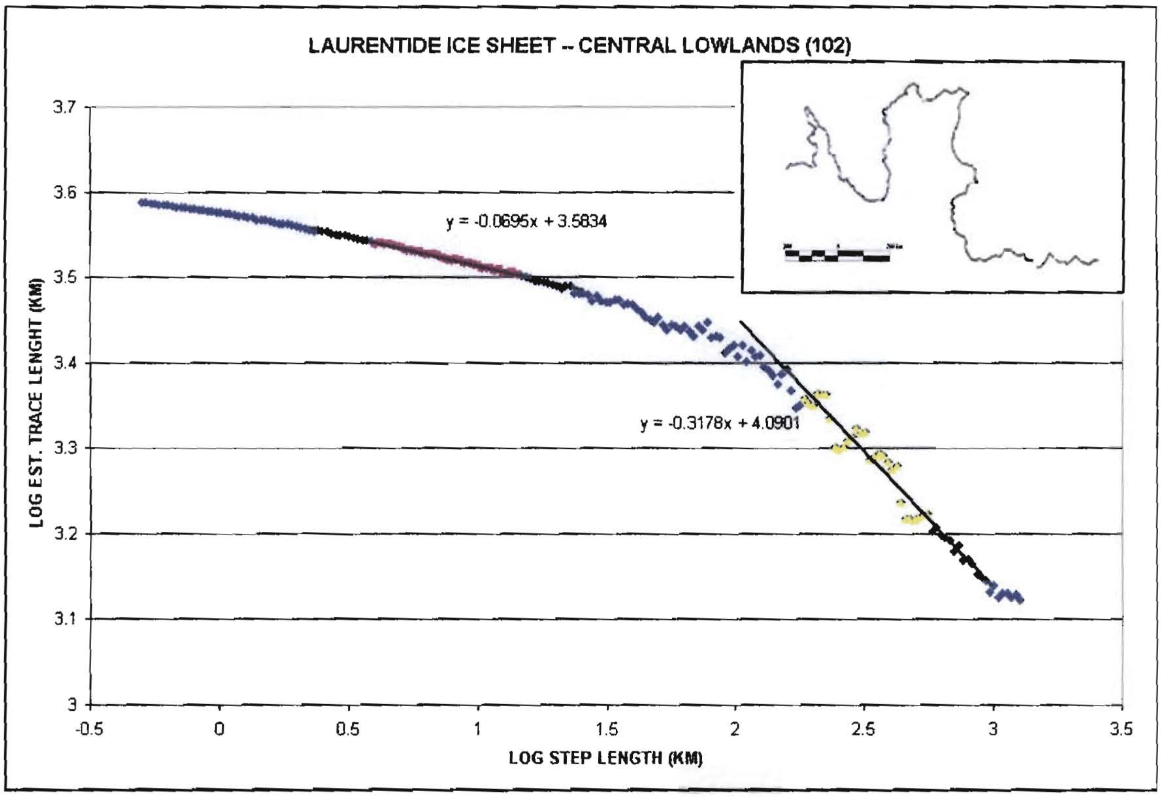


Figure 4. Richardson plot of the Laurentide Ice Sheet glacial maximum within the Central Lowlands Province.

Most of he smaller segments of the Central Lowlands (Figure 5), exhibit very different form than that of the original plot. Of the 17 subdivisions: 10 have D values that cross both scales (302, 303, 304, 305, 306, 308, 309, 312 315, 317), 3 have completely different fine and coarse scale D values (301, 307, 313), 2 have fine D (310, 314) and 2 has coarse D values (311, 316). The ranges of these segments are slightly smaller than those of the Great Plains due to a decrease in coarse D values. The range of the fine scale D values is 1.01-1.13 and the coarse scale range is 1.01-1.12.

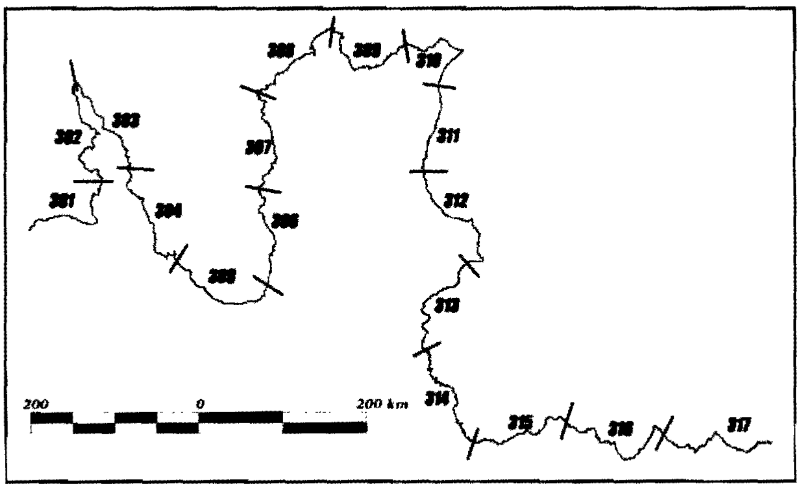


Figure 5. Subdivisions of the Central Lowlands Province.

The Appalachian Plateau Province

The plot from the Appalachian Plateau (Figure 6) has two distinct linear segments that are found in both scales. These yield D values of 1.13 and 1.07 respectively. In this case, the fractal dimension decreases as the step length increases.

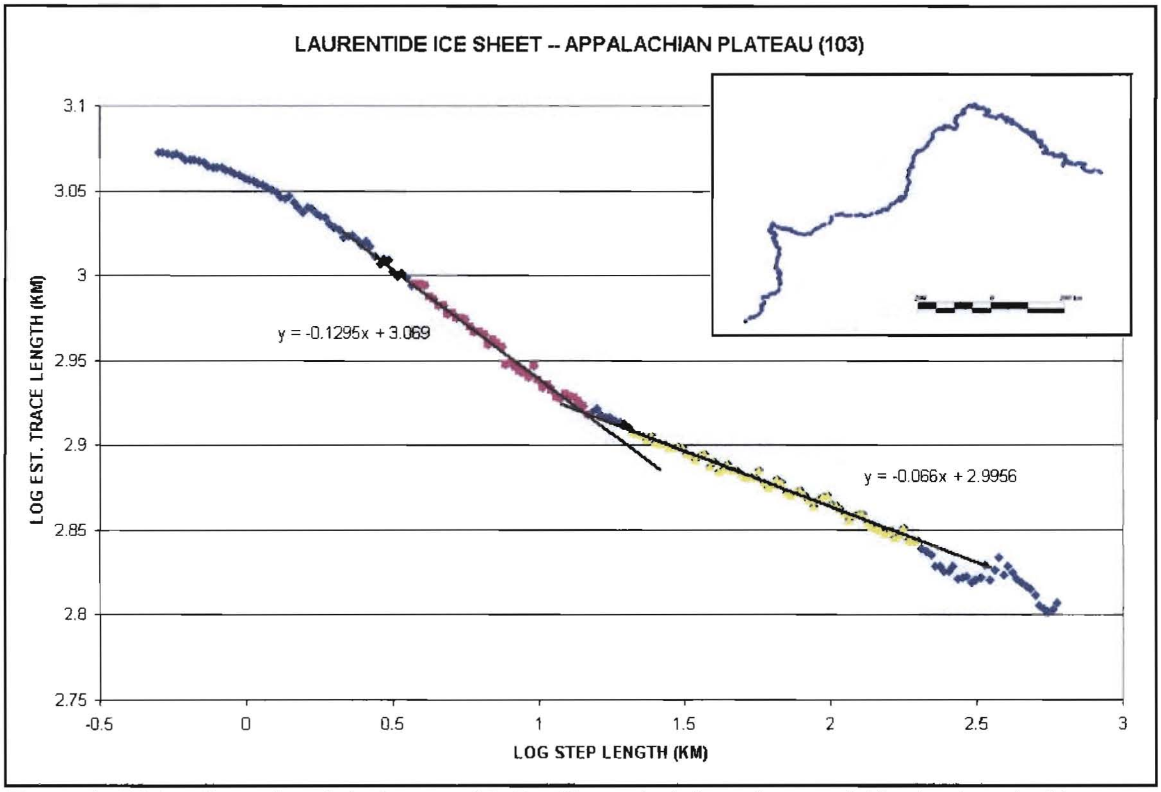


Figure 6. Richardson plot of the Laurentide Ice Sheet glacial maximum within the Appalachian Plateau Province.

The general form of this plot is very similar to the smaller sections (Figure 7) of the Appalachian Plateau Province. Three of the 4 sections show the same trend of decreasing D with increasing step length (401, 402, 403). The fourth segment reveals just one linear portion in the fine scales (404). The D range of the fine scales is 1.09-1.13 and the range of the coarse scales is 1.03-1.20.

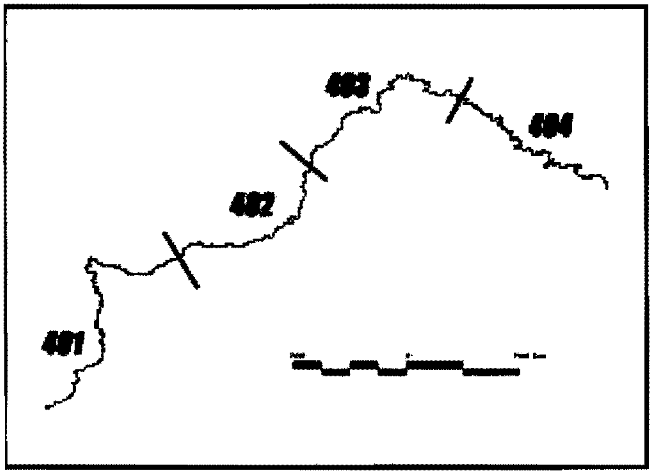


Figure 7. Subdivisions of the Appalachian Plateau Province.

The Valley and Ridge Province

There is only one linear segment on the Richardson plot of the Valley and Ridge Province (Figure 8). This D value (1.09) is found in the fine scales. This province has no subdivisions for comparison.

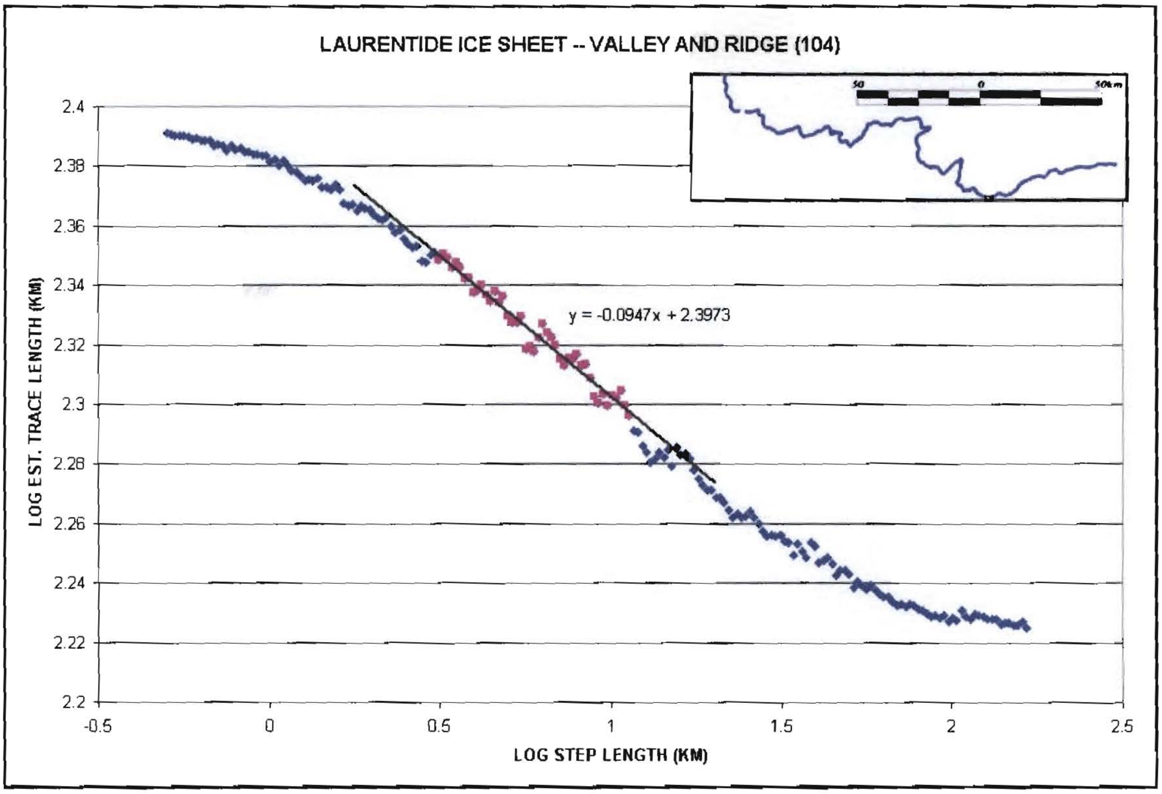


Figure 8. Richardson plot of the Laurentide Ice Sheet glacial maximum within the Valley and Ridge Province.

The Piedmont Province

The Piedmont Province (Figure 9) only has one fractal dimension (1.04) that lies in the fine scales. There are no subdivisions of this province.

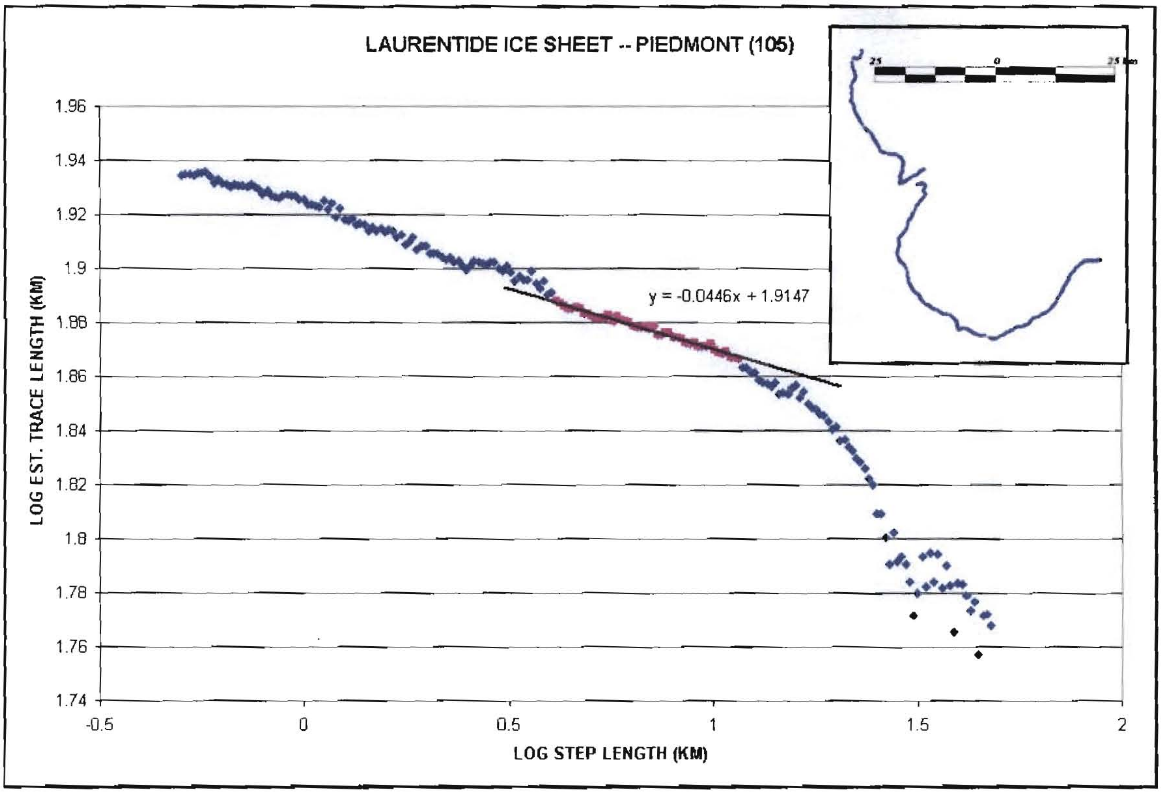


Figure 9. Richardson plot of the Laurentide Ice Sheet glacial maximum within the Piedmont Province.

Discussion

All segments of the Wisconsinan Maximum Glacial Advance Boundary show evidence of fractal character. Richardson plots display at least one area of linear tendency in the trace of each boundary segment. The vast majority of the plots possess D values in fine and coarse scales. This discussion will focus on several points: scale and step length range, D value range, comparison of subdivisions to the larger whole segment and comparison between provinces. Please refer to Appendix 2 for all Richardson plot data.

The comparison of step length ranges will reveal information regarding a shared process in the formation of the geometries of these boundary segments. Similar ranges of fractal character may indicate that an active scale-specific control is responsible the shaping of the boundary segments where this trend is present. This scale-specific control could be things such as: drainage patterns, ice dynamics of the glacier or local geologic landforms (geomorphology).

The actual D values are a means of comparison for the amount of wandering or wiggliness that a trace exhibits. This will lead to further possibilities for comparison of individual areas of fractal character. The comparison of the subdivisions will show how the sections represent the character and form of the original whole province. This is important to do because no similar previous work has been done in this field to allow for comparison.

Whole Province Sections

The Richardson plots of the Whole Province Sections are quite different in appearance and statistical output. Four of the boundary segments were found to have fractal character in the finer scales and only 3 displayed fractal character in the coarse scales. The fine scale fractal character found within the Central Lowlands Province has been placed on a quasi-linear series of points. This line is not perfectly straight and does exhibit some level of curvature. Though this piece of data is interesting and important to note for comparisons of general degrees of boundary trace wandering between provinces; it may not be as significant as other fractal dimensions. This is important to note in future references.

The limited number of coarse scale values may be attributed to the short length of some of the segments (i.e. the Valley and Ridge and the Piedmont Provinces). Segments that are shorter in length will have a smaller range of step lengths that were used to measure the line. Therefore, these segments will lack data for the larger steps. The definition of the range of the scales is based on resolution of the map and is not proportional to the length of the segment in question. When data is not available for larger step lengths there can be no coarse scale fractal character. With this in mind, it can be noted that the range of the fine scale fractal dimension is significantly limited while the coarse scale range can be vast depending on the overall length of the line in question.

The range of the D values from these segments is quite wide. The fine scales include D values of 1.04-1.13. The coarse scale range is even larger with values of 1.07-1.32.

The Central Lowlands Province (Figure 4) displays an opposite form from that of the Appalachian Plateau Province (Figure 6). The Central Lowlands plot has heterogeneous wandering as it shows two cases of self-similar character, one in the fine and coarse scales. The Appalachian Plateau has a similar pattern. However, the Lowlands has less wandering at fine scales and more wandering at coarse scales. The Plateau section has greater wandering in the fine scales than the coarse scales.

There are linear segments that appear in the finer scales of several of these plots. However, this data might be contaminated by false smoothing of the boundary do to poor resolution of the maps from which the data was gathered. These segments have thus been omitted from this study.

Figure 10 shows the relationship of D values with the range of step lengths over which these values exist. Note that 4 of the 5 provinces have fine scale fractal dimensions that nearly span the same step lengths. This points out that there are small scale fractal features on these line segments that have similar size. However, the actual D values for these sections vary widely. The higher fractal dimensions imply higher degrees of wandering. This means that the small scale features that these segments have in common are of different form. From Figure 10, it is also apparent that the three coarse scale fractal dimensions are not very similar in range or value.

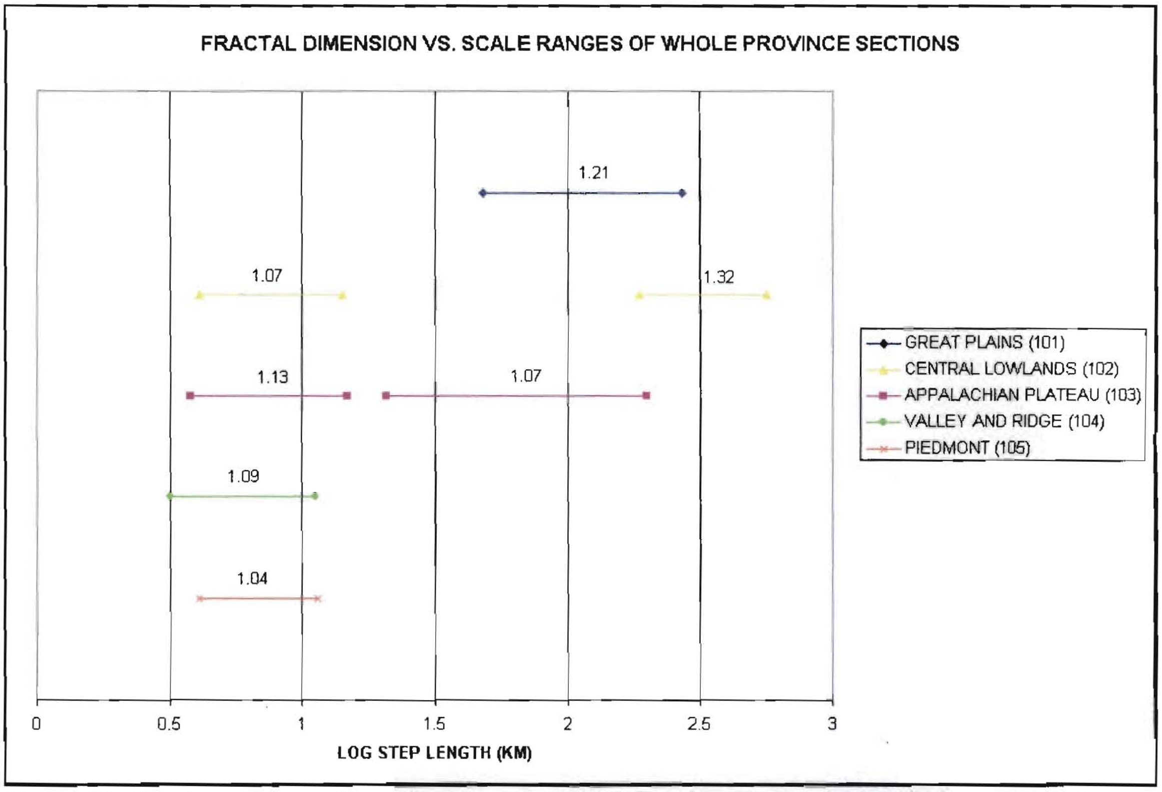
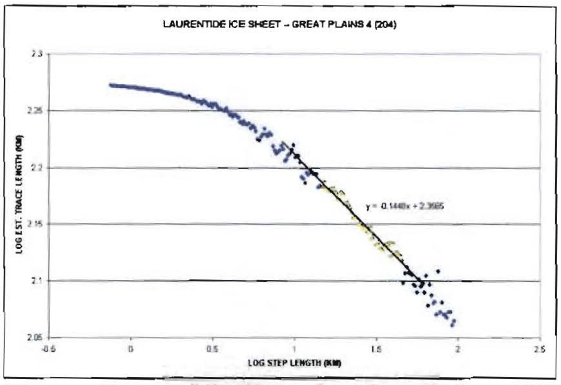


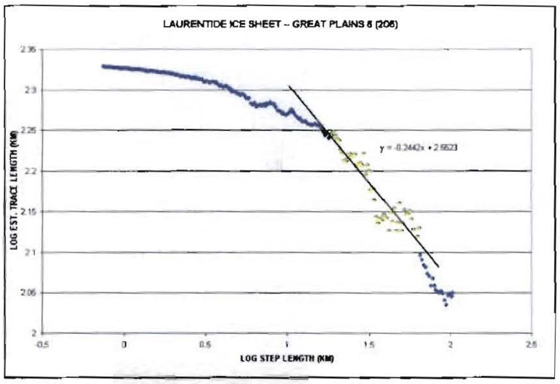
Figure 10. This figure shows the scale ranges for which D values were calculated for each province dataset.

The Great Plains Sections

Most of the 11 plots from the Great Plains Province show comparable form with that of the original segment; this being a general concave down shape. Some of the plots appear sigmoidal. Plots such as 204 (Figure 11A) and 206 (Figure 11B) appear very similar the Great Plains plot (Figure 2). Plots such as 205 and 206 even have similar D values in the coarse scales. The ranges of step lengths between the original and the subdivisions are quite different.

This difference comes down to a matter of relative segment length. The maximum length of the Great Plains segment, as calculated by the divider analysis is approximately 2375 km, while the maximum length of section 206 is approximately 213 km. This is a difference of an order of magnitude. Therefore, the divider analysis will be able to use far larger step lengths when performing the calculations. In doing so there will be a greater possibility of finding fractal character in scales that can not be found on the plot of a subsection.





Figures 11 A and B. Plots of sections 204 and 206 from the Great Plains Province. Note the similarity with plot 101 (Figure 2).

The ranges of the D values from the Great Plains sections are greater than the ranges of any other province; 1.01-1.17 in the fine scales and 1.05-1.24 in the coarse. The average of each scale was 1.10 and 1.14 respectively. Therefore, on average, fractal dimensions increase with step length. Fractal character is also more prominent in the coarse scales than in the finer scales. This may be indicative of some large scale feature or process that is causing self-similarity. Figure 12 shows that there is no apparent correlation of these coarse scale fractal dimensions, in step length range or D value. Figure 12 also shows a clustering of fractal character in the 0.80 to 1.60 (approximately 5-40 km) step lengths. Overall, these subdivisions moderately accurately represent the form and character of the larger whole province section.

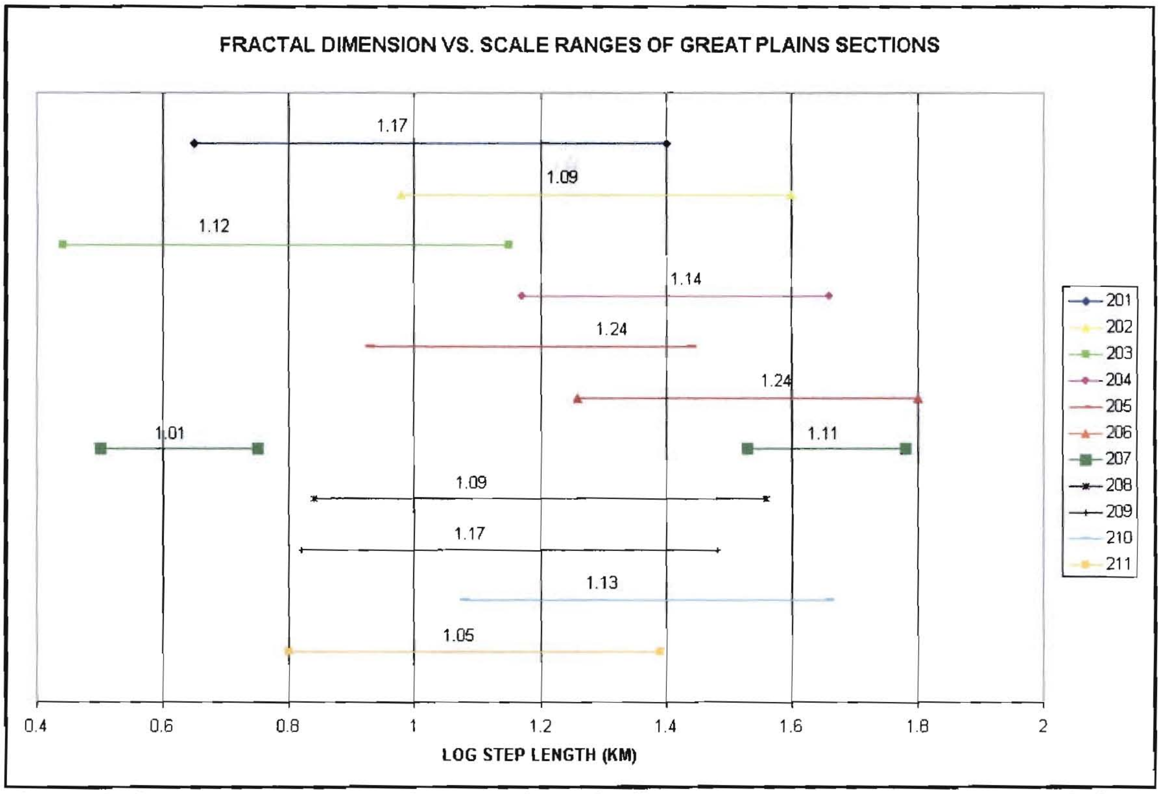


Figure 12. This figure shows the scale ranges for which D values were calculated for each subdivision of the Great Plains Province.

The Central Lowlands Sections

The vast majority of the sections from the Central Lowlands do not show a form similar to the original whole province segment. The original plot (Figure 4) displays two areas of fractal character with increased wandering in the coarse scales. This trend is only seen in one subsection (301). Fourteen of the seventeen sections in the Central Lowlands have plots that show one area of linearity. Ten of these sections have fractal character that crosses fine and coarse scales. This bridging of fine and coarse scales represents homogeneous fractal character within the plots. These sections exhibit a great range of step length as can be seen in Figure 13. This figure also makes evident a general lack of uniformity or pattern in regards to a relationship between the step length scale and D value. There is a grouping or tight packing of fractal character areas between the plots;

however, the range of step lengths of the grouping is quite large. There is no neat organization such as in Figure 10.

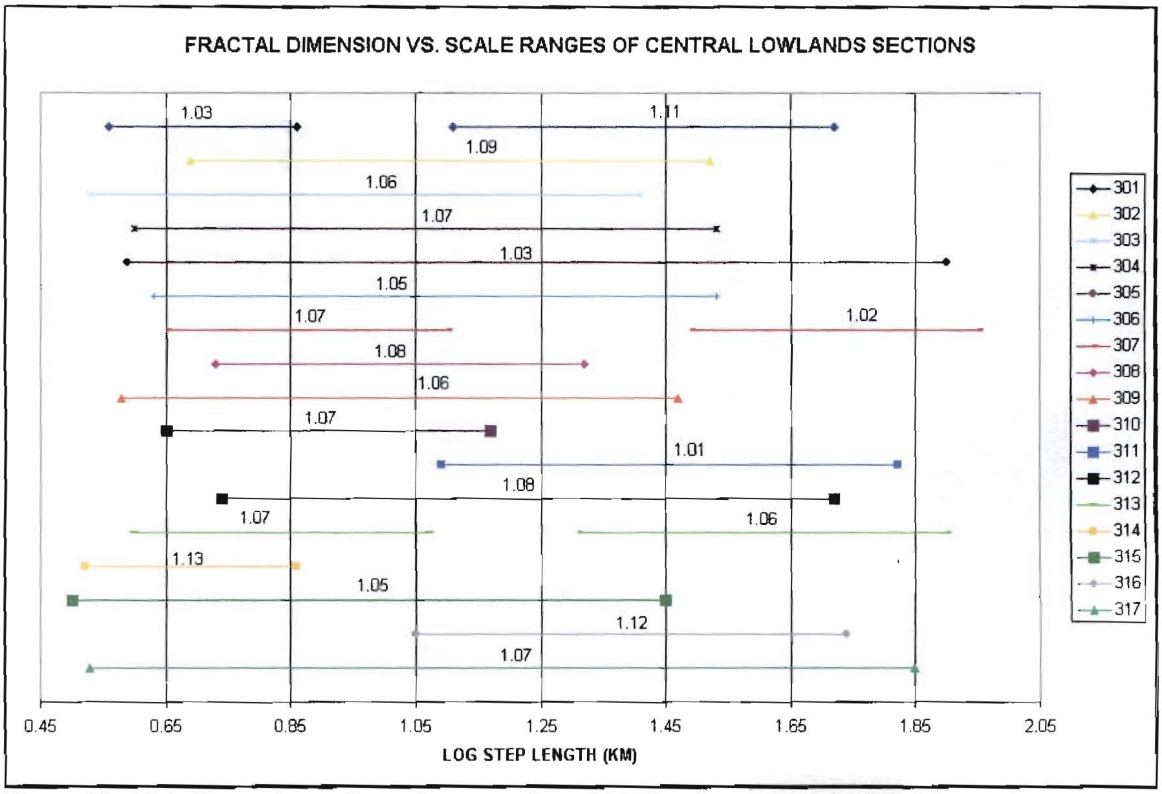
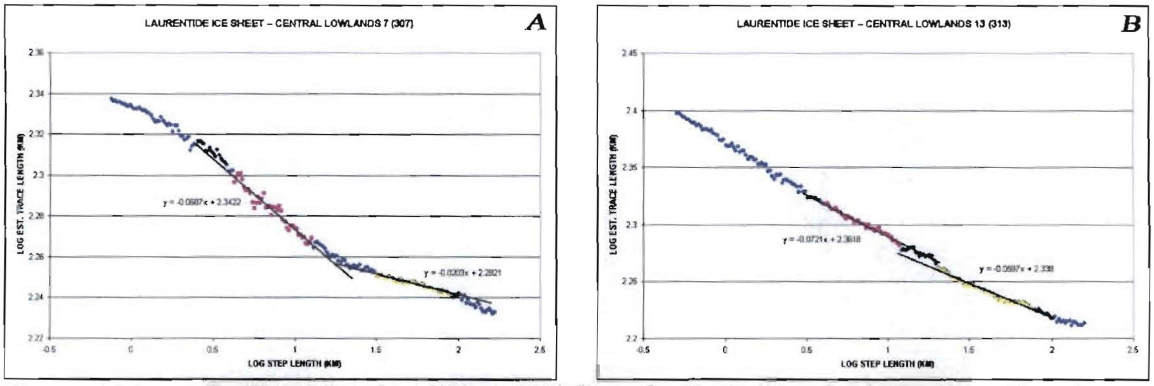


Figure 13. This figure shows the scale ranges for which D values were calculated for each subdivision of the Central Lowlands Province.

Two sections from the Central Lowlands Province show an opposite trend from that of the original whole province plot. Plots 307 (Figure 14A) and 313 (Figure 14B) show two separate sections that possess greater wandering at coarser scales. The ranges of D values are smaller in this province. Fine scale values run from 1.03-1.13 and coarse scale values go from 1.01-1.12. The averages of these two scales were also very close, 1.07 and 1.06 respectively. All the plots from this segment do display significant fractal character but are not very representative of the greater whole of the province.



Figures 14 A and B. Plots of sections 307 and 313 from the Central Lowlands Province. Note the difference from plot 102 (Figure 4).

The Appalachian Plateau Sections

All four of the subsection plots of the Appalachian Plateau Province show similar form to that of the original province segment. They show a general concave up form and all but one show two areas of linearity. The fourth pot (404) has only one area of fractal character in the fine scale. A linear segment in the coarse scales may be seen but for reasons previously discussed it is not considered in this study. Section 403 (Figure 15) shows nearly identical form and D values as section 103 (Figure 6). The only large difference is the range of step lengths at which the D values occur, which is again an issue with the overall relative lengths of the sections. It would be expected that the ranges of D values would be lower than other provinces due to the fewer amount of sections. This is true for the fine scales which have a range of 1.09-1.13. However, the range of the coarse scales is quite larger: 1.03-1.20. The averages are very similar with 1.11 for the fine and 1.10 for the coarse scale. The subsections of the Appalachian Plateau Province seem to better represent the larger province boundary, from which it was derived, than any other subdivisions.

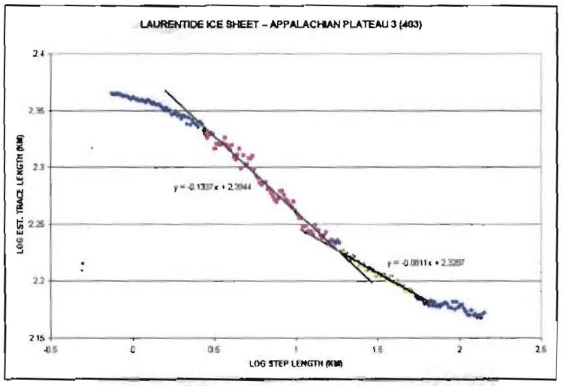


Figure 15. Plot of section 403 from the Appalachian Plateau Province. Note the similarity from plot 103 (Figure 6).

By looking at Figure 16 one can easily see that there is a nice alignment of the fractal character segments. This denotes similar step lengths at which the fractal dimensions are exhibited. This trend can be seen in the finer and coarser scales. This is a very interesting quality that may allude to a scale-specific process acting on these boundaries that has controlled their geometries. There may be two separate processes that are active in these areas (one at fine and coarse scales) or it can be one process that can affect the shape of the boundary at smaller and larger scales.

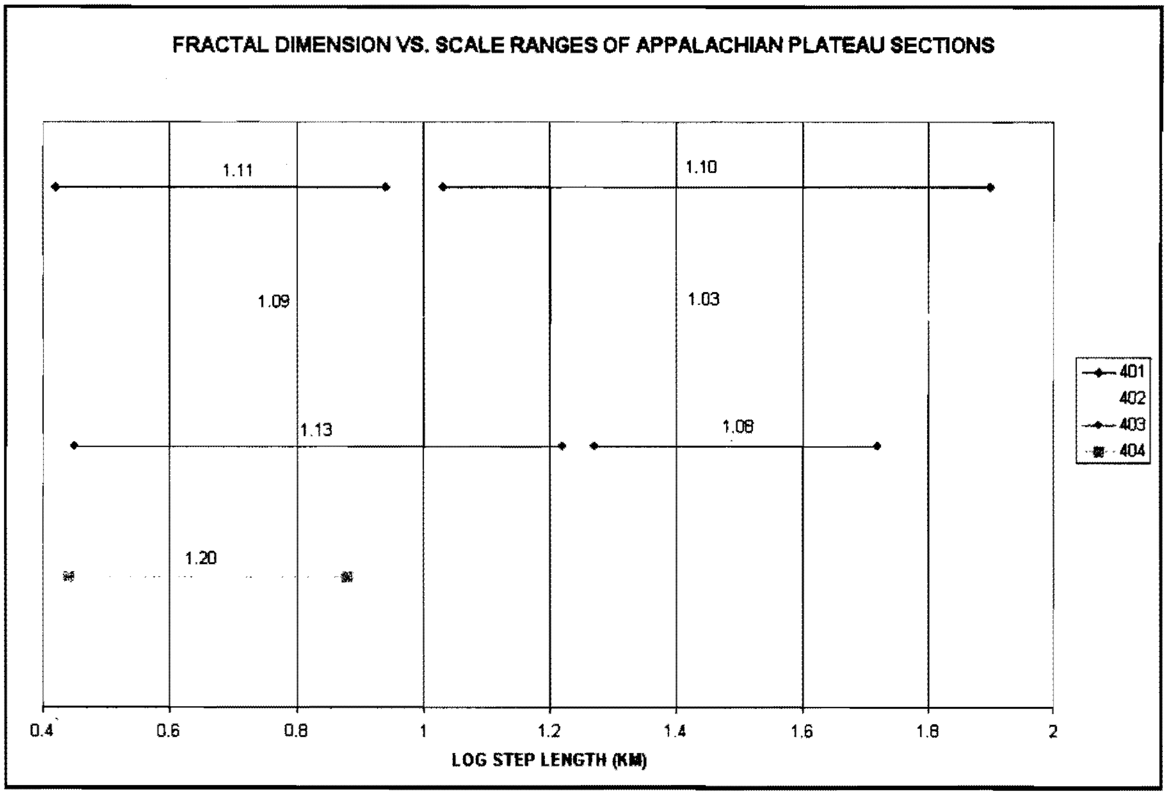


Figure 16. This figure shows the scale ranges for which D values were calculated for each subdivision of the Appalachian Plateau Province.

Conclusions

We have shown that glacial boundaries do express fractal character. Each of the province-delimited segments and subsequent divisions is characterized by at least one D value. Some plots showed two areas of linearity while others only exhibited one. These linear segments were found at fine (3-10 km) and coarse scales (10-1000 km); and some cases crossed both scales. The D range at coarse scales is 1.01-1.17 and the range at coarse scales is 1.01-1.32.

When comparing sections from differing provinces a few conclusions can be drawn. No province shows D values that are specifically unique unto itself. The highest D value is (1.32) located in the coarse scale of the Central Lowlands Province (whole province segment). However, the highest value of any subsection of this province is only 1.20. The Great Plains Province subsections have a high D of 1.24. This does not show evidence that high fractal dimensions are associated with any one province. Very low D values (1.01-1.04) are also found in all provinces (except for the Valley and Ridge province which only has one fractal dimension).

The averages of the D values found in the Great Plains sections (1.10 in the fine scales and 1.14 in the coarse scales) are larger than that of the average of all sections (1.08 in the fine scales and 1.11 in coarse scales). Conversely, the averages of the D values of the Central Lowlands sections are below the overall average with values of 1.07 in the fine scales and 1.06 in the coarse scales.

The two provinces that were not split into subsegments, the Valley and Ridge and Piedmont Provinces, show only one fractal dimension in the fine scales; 1.09 and 1.04 respectively. These values fit nicely into the overall range of D values at this scale.

However, the D value of the Valley and Ridge Province (1.09) is higher than the overall average and the Piedmont Province D value (1.04) is lower than the average.

There does not seem to be any trend in the data associated with an increase in topographic relief of the terrain within the provinces. There is also no set range of step length of D values that exists in all province sections that would prove the existence of one universal active scale-specific mechanism was involved in the formation of the geometries of these boundaries.

The data from the Appalachian Plateau Province is of significant interest. The plot form of this section displays two areas of fractal character. However, unlike most other segments this plot shows less wandering the coarse scales. The subsections of this province showed excellent correlation with that of the larger whole segment. These smaller sections also showed similar range of step lengths at which fractal character was observed. This data could represent the workings of a scale-specific mechanism on these boundary segments, which was the ultimate goal of this project. Further work in this area would be well warranted.

Acknowledgments

The research that this paper is based on was graciously supported by the Ball State University Honors College and the Ball State University Office of Academic Research and Sponsored Programs. CAD work and other technical assistance was provided by Mike Kutis.

References

Bishop, Michael P., 1998. Scale-dependant analysis of satellite imagery for characterization of glacier surfaces in the Karakoram Himalaya. Geomorphology, 21: 217-232.

Elliot, Joanne K., 1989. An investigation of the change in surface roughness through time on the foreland of Austre Okstindreen, North Norway. Computers and Geosciences, 15: 209-217.

Fenneman, Nevin M., 1928. Physiographic divisions of the United States; third edition, revised and enlarged. Annals of the Association of American Geographers, 18: 261-353.

Mandlebrot, B. B., 1967. How long is the coast of Britain? – Statistical self-similarity and fractional dimension. Science, 156: 636-638.

Mandlebrot, B. B., 1983. The Fractal Geometry of Nature. Freeman & Co., New York.

McClelland, S., 1985. Using fractals to measure the progress of erosion. Proceedings of the West Virginia Academy of Science, 57: 91-95.

Nikora, V. I., 1991. Fractal structures of river plan forms. Water Resources Research, 27: 1327-1333.

Snow, R. Scott., 1989. Fractal sinuosity of stream channels. Pure and Applied Geophysics, 131: 99-109.

Snow, R. Scott & Lowell, Thomas V and Robin F. Rupp., 1991. A Field Guide: The Kelley's Island Glacial Grooves, subglacial features on the Marblehead Peninsula, carbonate petrology, and associated paleontology. Ohio Journal of Science, 91: 16-26.

Snow, R. Scott & Mayer, L., 1992. Introduction. Geomorphology Special Edition,1-4.

Snow, R. Scott., 1998. Fractal characteristics of drainage basin boundaries in Puerto Rico. International Association of Hydrological Sciences, 253: 195-201.

Szekely, Balazs., 1999. Results of fractal dimensions applied on the digital elevation model of the Eastern Alps. Geologie, Palaeontologie, Stratigraphie, 52: 212-222.

Appendix 1. Map Listings

Geologic map of the 1°x2° Cincinnati Quadrangle, Indiana and Ohio, showing bedrock and unconsolidated deposits. Map. Indianapolis: Department of Natural Resources, 1972.

Geologic map of the 1°x2° Indianapolis Quadrangle, Indiana and Illinois, showing bedrock and unconsolidated deposits. Map. Indianapolis: Department of Conservation, 1961.

Map showing the thickness and character of the Quaternary sediments in the glaciated United States east of the Rocky Mountains—northern plains states (west of 102° west longitude). Map. Denver: U.S. Geological Survey, 1994.

Quaternary geologic map of the Chicago 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1983.

Quaternary geologic map of the Dakotas 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1995.

Quaternary geologic map of the Des Moines 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1991.

Quaternary geologic map of the Hudson River 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1992.

Quaternary geologic map of the Lake Erie 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1991.

Quaternary geologic map of the Lake Superior 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1984.

Quaternary geologic map of the Minneapolis 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1983.

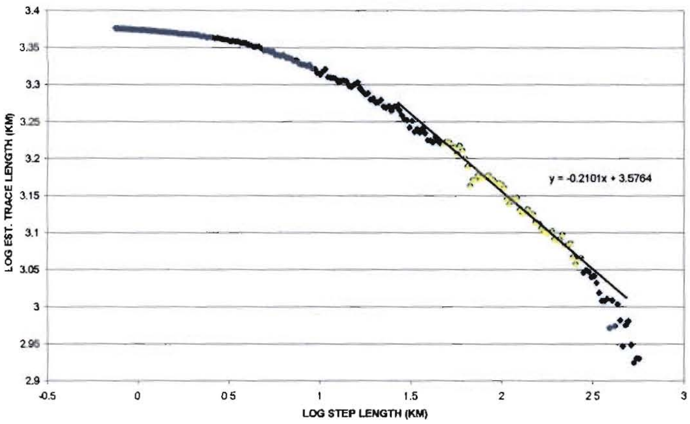
Quaternary geologic map of the Platte River 4 °x6 ° Quadrangle, United States. Map. Denver: U.S. Geological Survey, 1994.

Quaternary geology of Ohio. Map. Columbus: Ohio Department of Natural Resources, 1999.

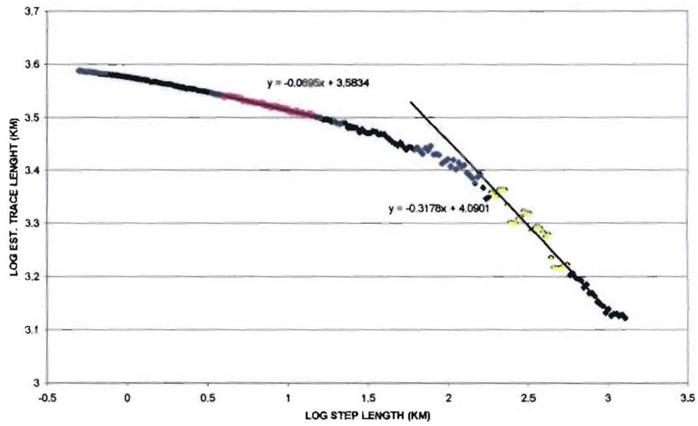
Surficial Deposits of Illinois. Map. Champaign: Illinois State Geological Survey, 2000.

APPENDIX 2A WHOLE PROVINCE SECTIONS [101-105]

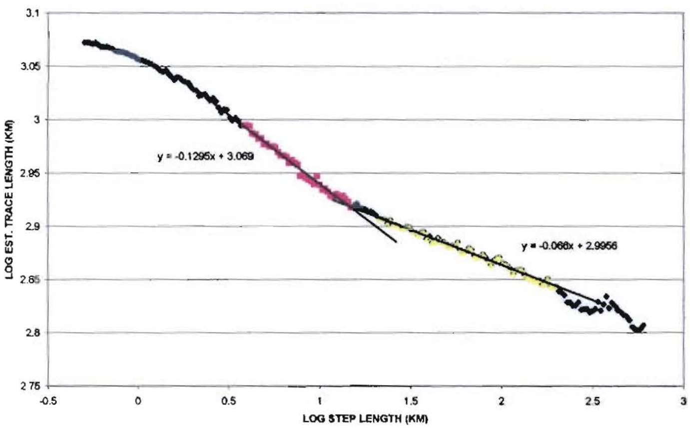
LAURENTIDE ICE SHEET – THE GREAT PLAINS PROVINCE (101)



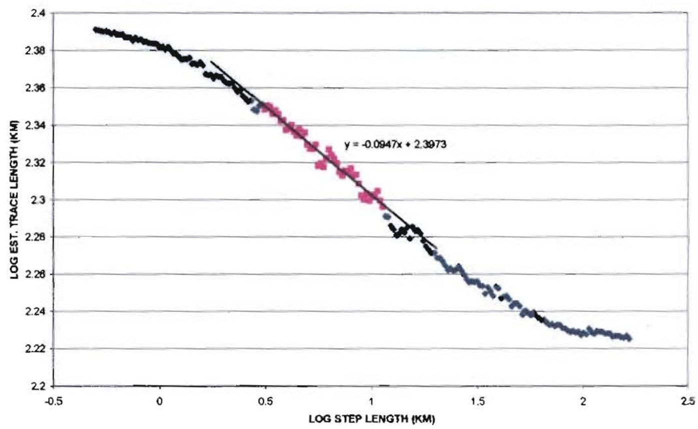
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS (102)



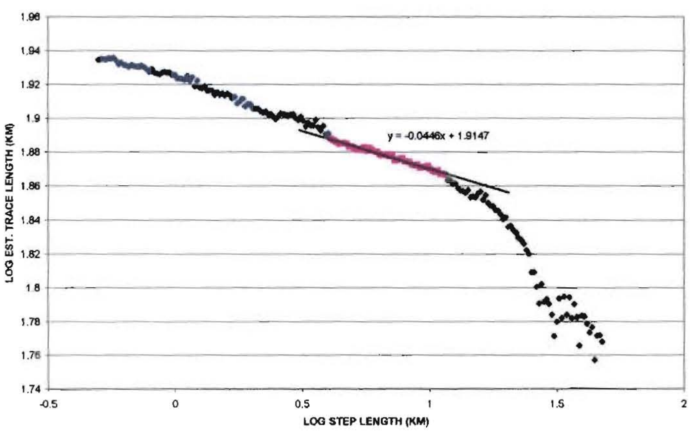
LAURENTIDE ICE SHEET – APPALACHIAN PLATEAU (103)



LAURENTIDE ICE SHEET – VALLEY AND RIDGE (104)

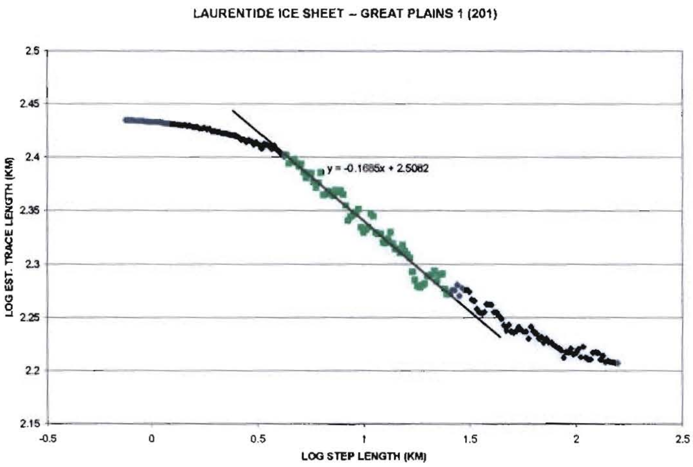


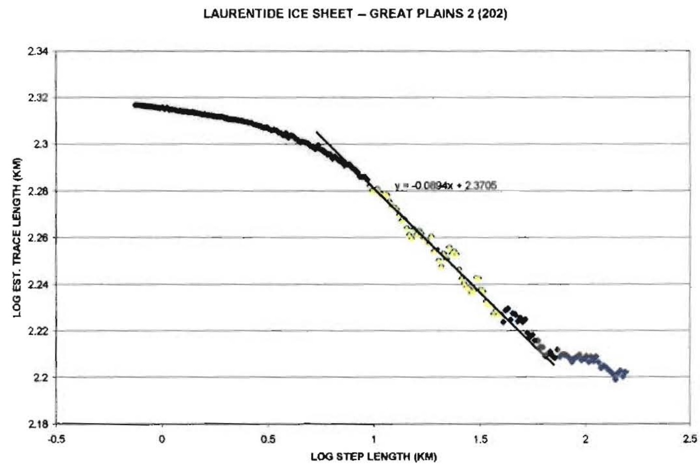
LAURENTIDE ICE SHEET – PIEDMONT (105)

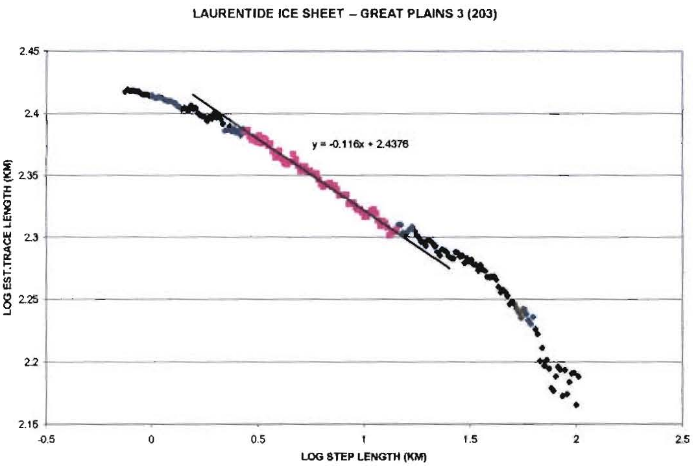


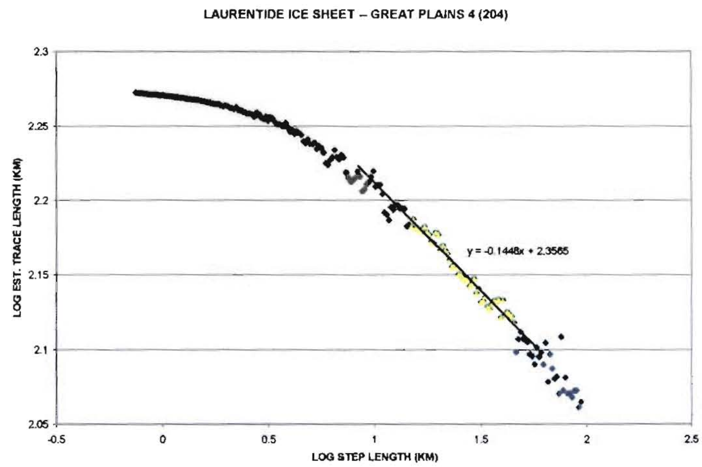
- FINE SCALES
- COARSE SCALES
- FINE AND COARSE SCALES

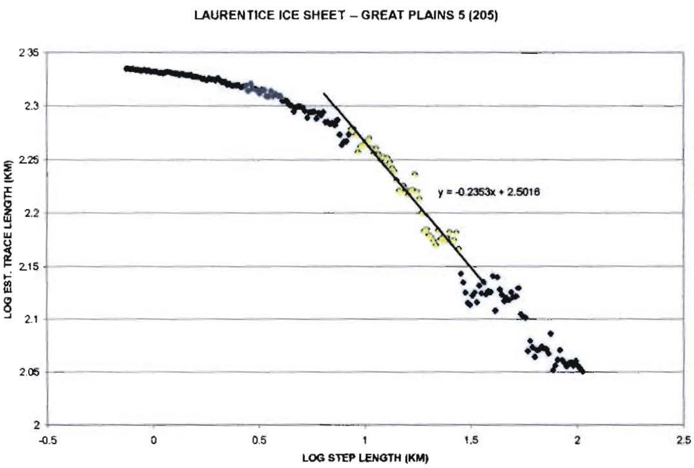
APPENDIX 2B GREAT PLAINS SECTIONS [201-206]

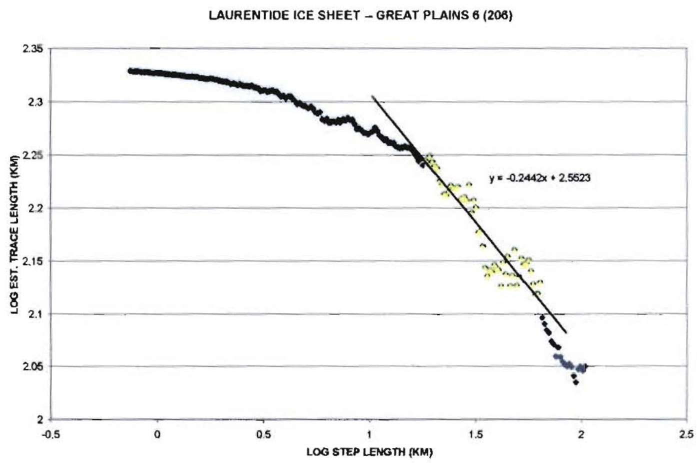




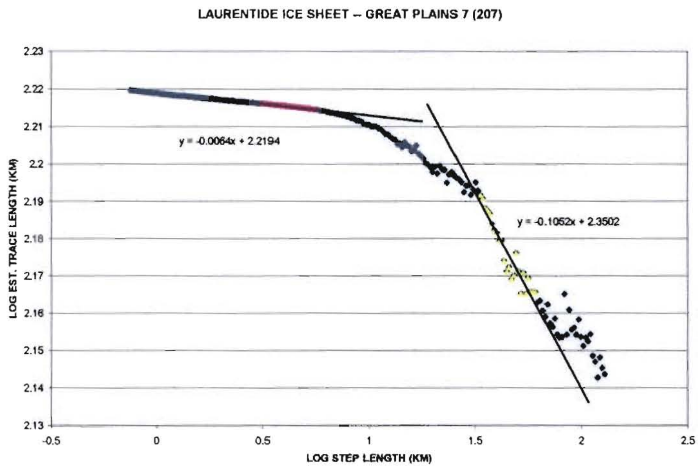


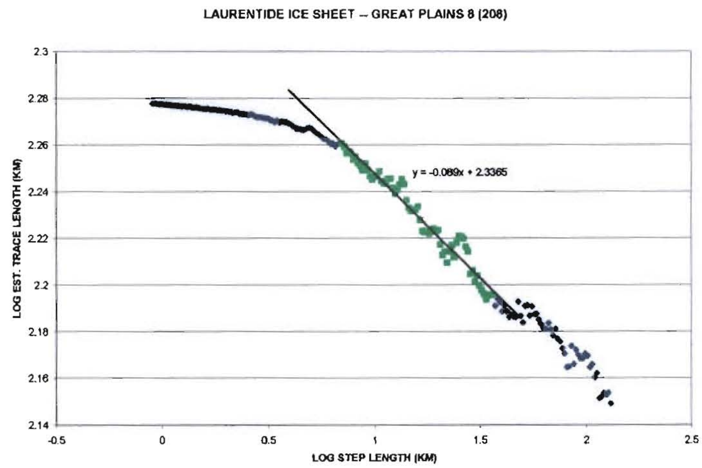


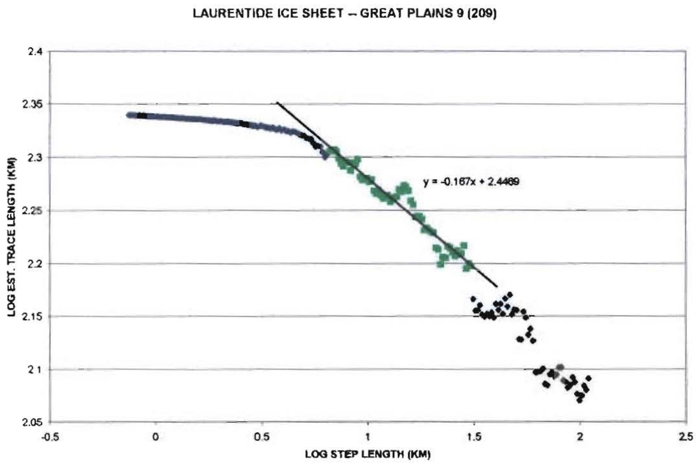


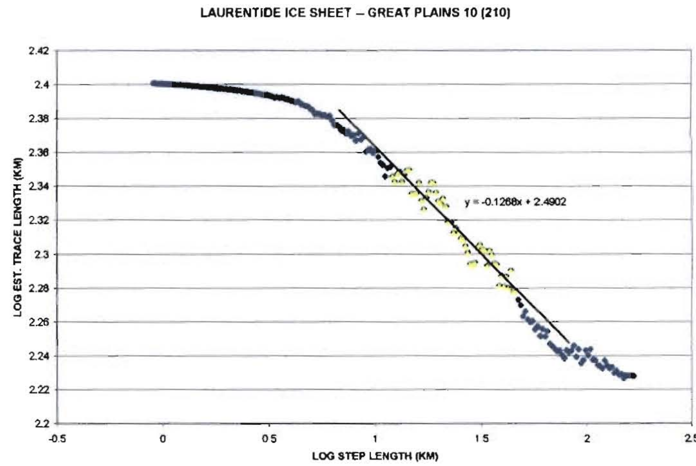


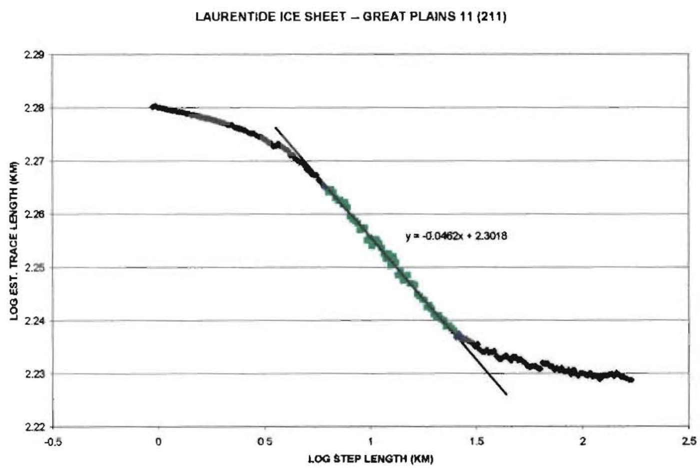
APPENDIX 2B GREAT PLAINS SECTIONS CONTINUED [207-211]

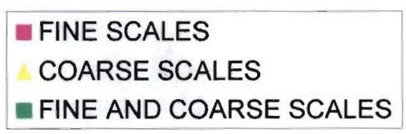






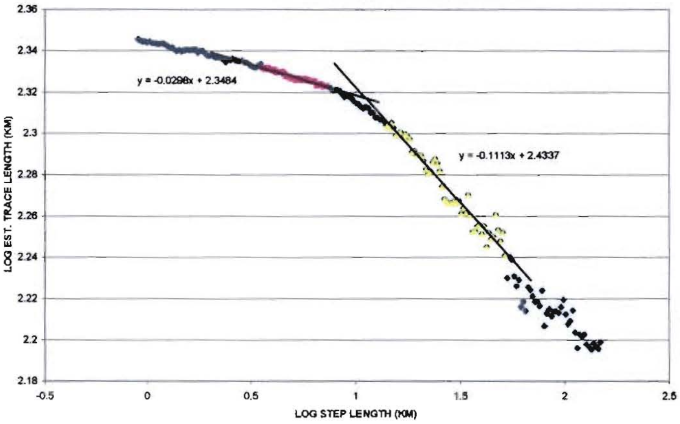




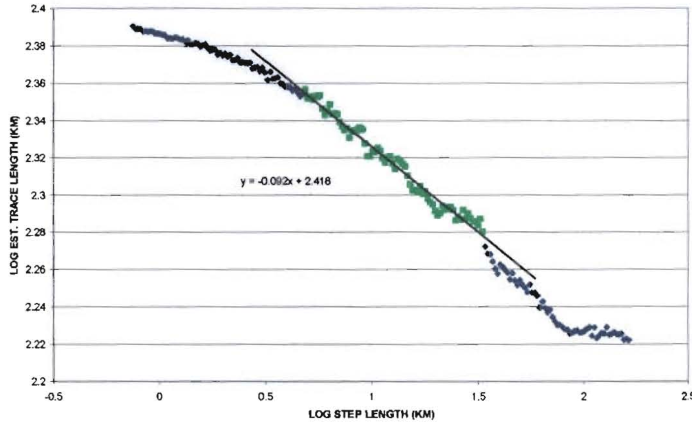


APPENDIX 2B CENTRAL LOWLANDS SECTIONS [301-306]

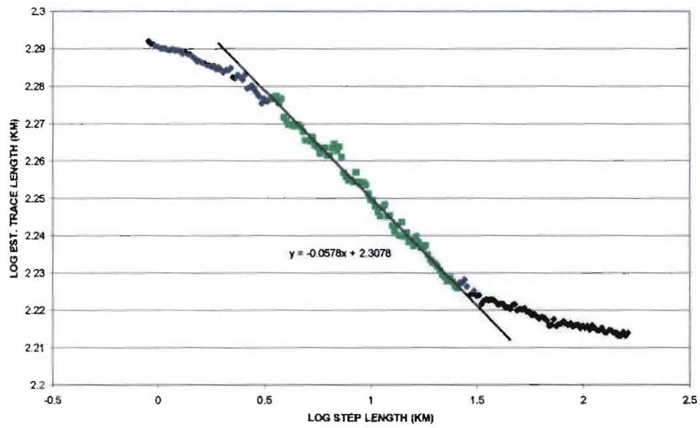
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 1 (301)



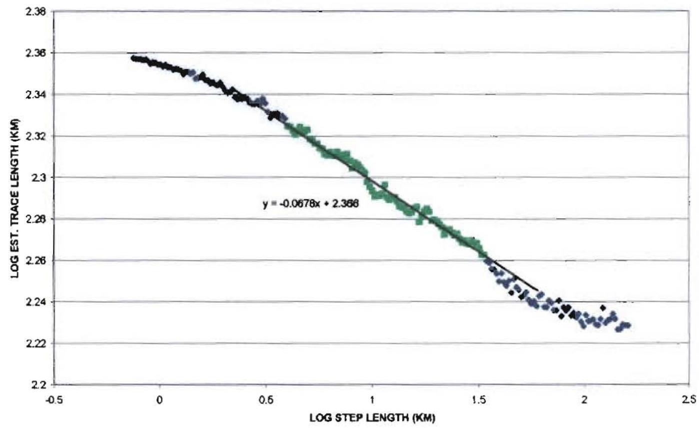
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 2 (302)



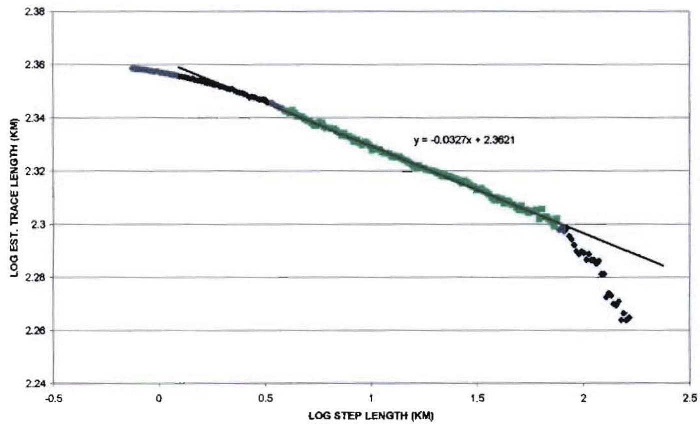
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 3 (303)



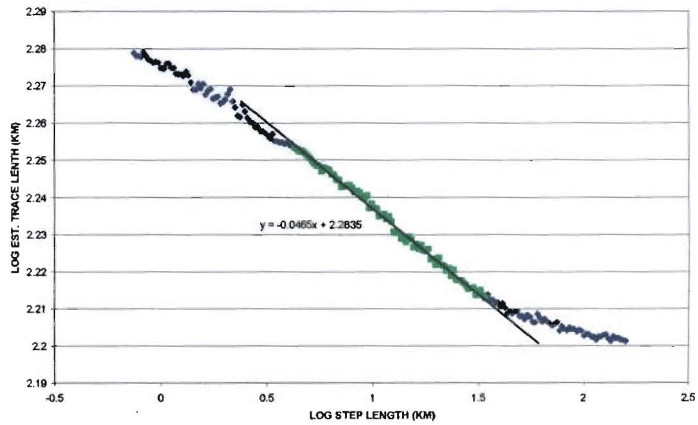
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 4 (304)



LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 5 (305)

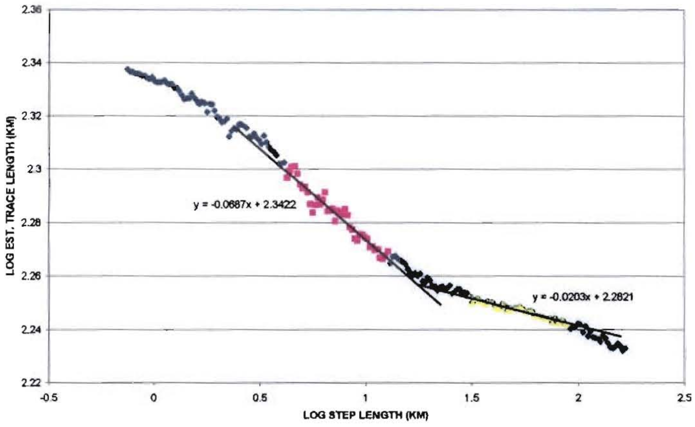


LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 6 (306)

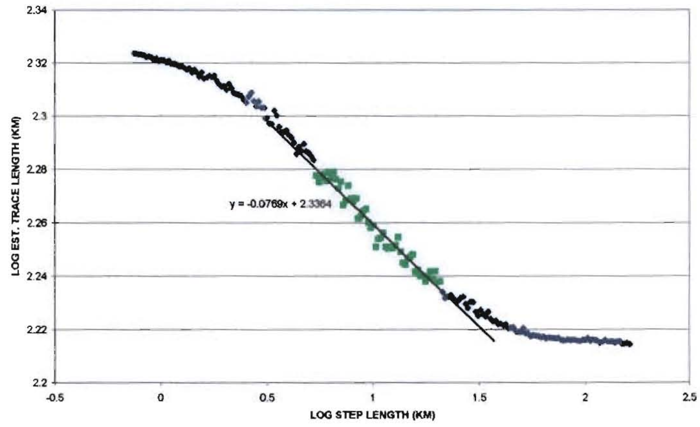


APPENDIX 2B CENTRAL LOWLANDS SECTIONS CONTINUED [307-312]

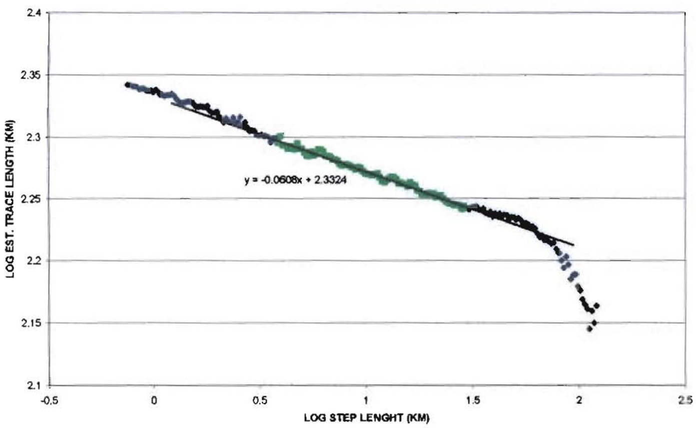
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 7 (307)



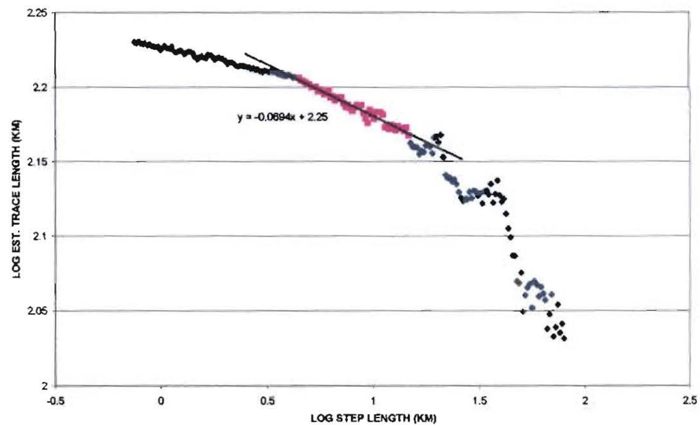
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 8 (308)



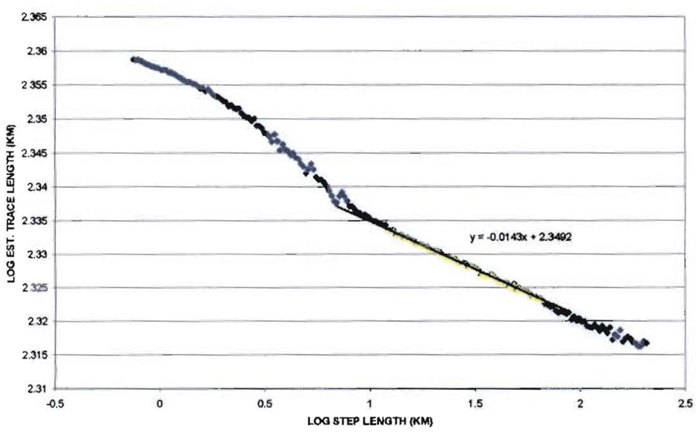
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 9 (309)



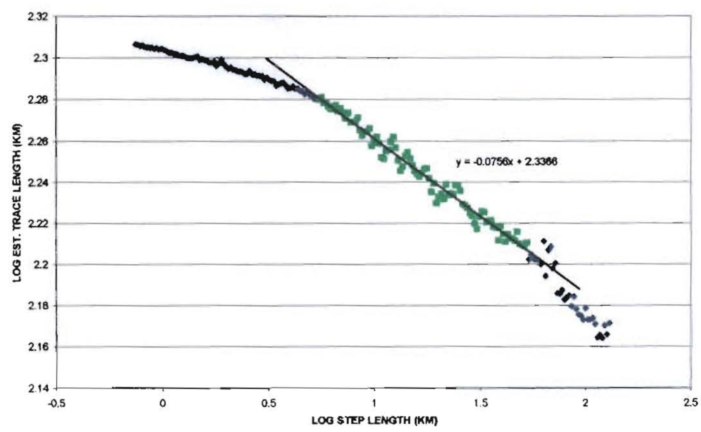
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 10 (310)



LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 11 (311)

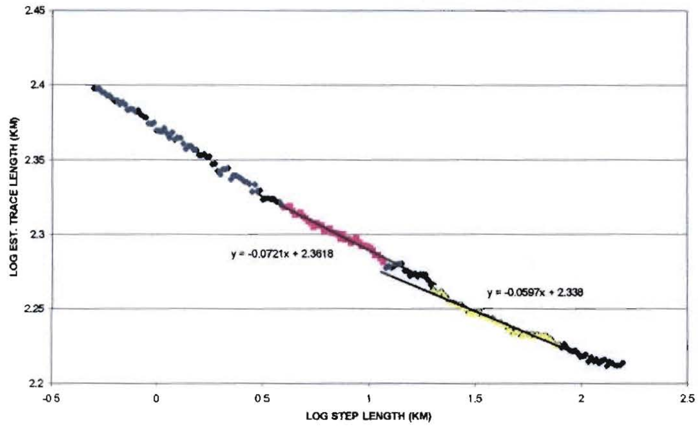


LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 12 (312)

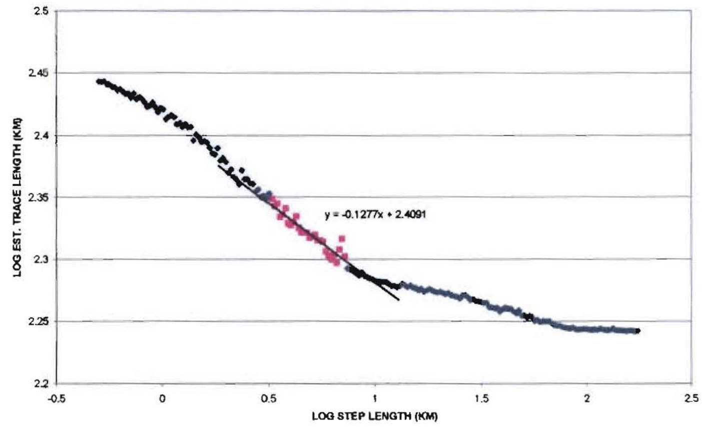


APPENDIX 2B CENTRAL LOWLANDS SECTIONS CONTINUED [313-317]

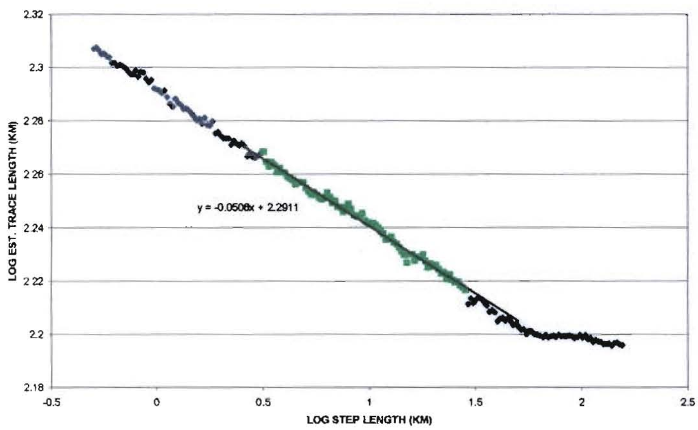
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 13 (313)



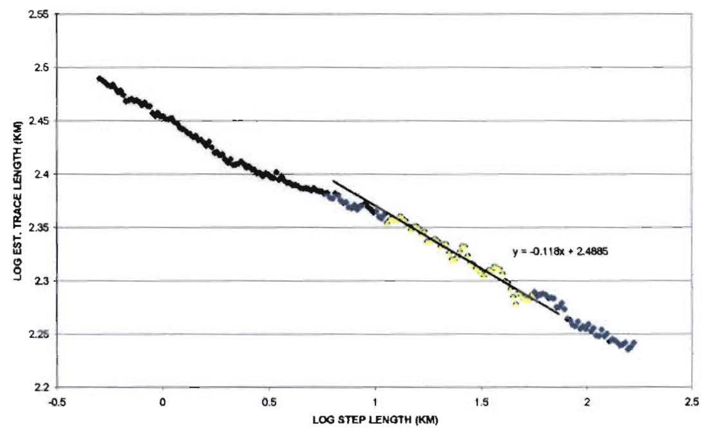
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 14 (314)



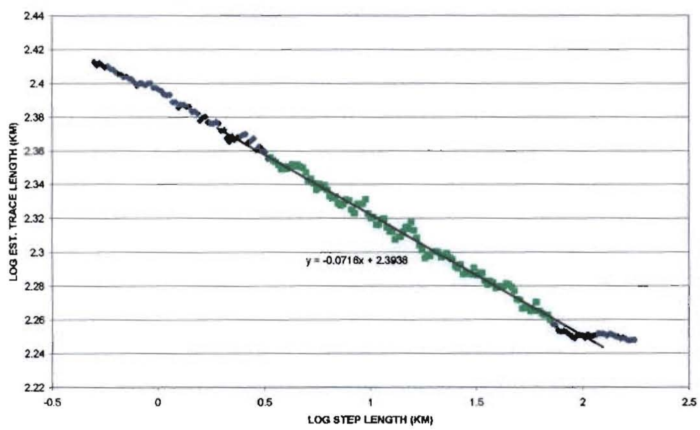
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 15 (315)



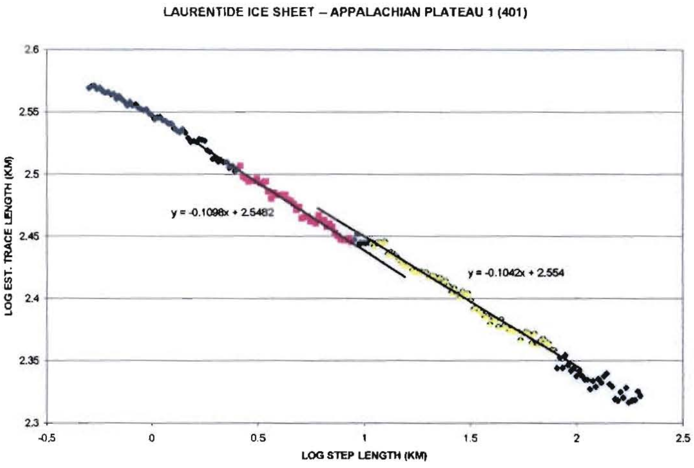
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 16 (316)

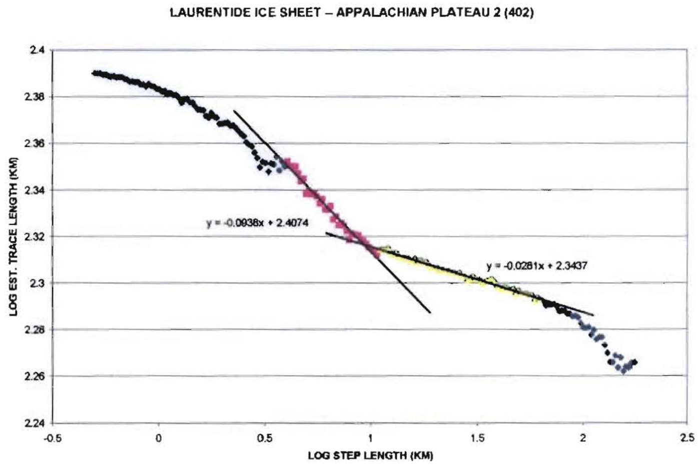


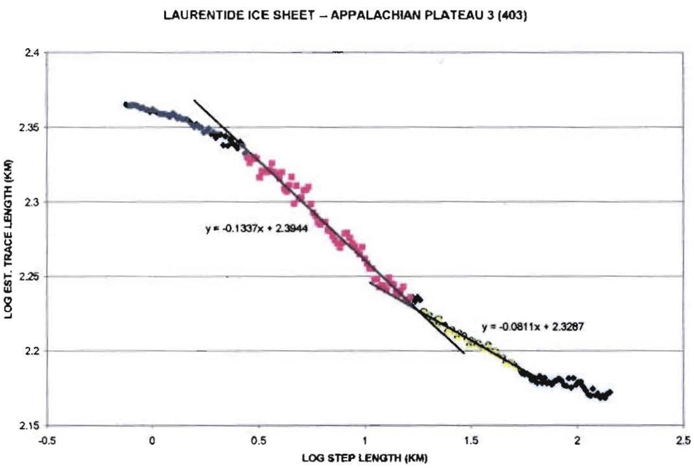
LAURENTIDE ICE SHEET – CENTRAL LOWLANDS 17 (317)

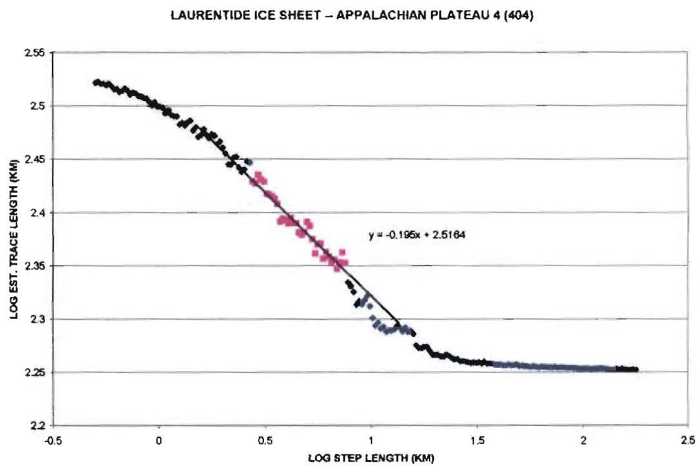


APPENDIX 2C APPALACHIAN PLATEAU SECTIONS [401-404]









FINE SCALES
COARSE SCALES
FINE AND COARSE SCALES

APPENDIX 3A						
STATISTICS OF WHOLE PROVINCE SECTIONS						
				RANGE	**(D)**	**NUMBER OF SECTIONS**
PROVINCE	**FINE (D)**	**COARSE (D)**				**INCLUDED IN RANGE**
GREAT PLAINS		1.21		FINE	1.04-1.13	4
CENTRAL LOWLANDS	1.07	1.32		COARSE	1.07-1.32	3
APPALACHIAN PLATEAU	1.13	1.07				
VALLEY AND RIDGE	1.09			**AVERAGE**	**(D)**	**NUMBER OF SECTIONS**
PIEDMONT	1.04					**INCLUDED IN AVERAGE**
				FINE	1.08	4
				COARSE	1.20	3

APPENDIX 3B

STATISTICS OF GREAT PLAINS SECTIONS

SECTION NUMBER	FINE (D)	COARSE (D)		RANGE	(D)	NUMBER OF SECTIONS
201	1.17	1.17				INCLUDED IN RANGE
202		1.09		FINE	1.01-1.17	6
203	1.12			COARSE	1.05-1.24	10
204		1.14				
205		1.24		AVERAGE	(D)	NUMBER OF SECTIONS
206		1.24				INCLUDED IN AVERAGE
207	1.01	1.11		FINE	1.10	6
208	1.09	1.09		COARSE	1.14	10
209	1.17	1.17				
210		1.13				
211	1.05	1.05				

APPENDIX 3C

STATISTICS OF CENTRAL LOWLANDS SECTIONS

SECTION NUMBER	FINE (D)	COARSE (D)		RANGE	(D)	NUMBER OF SECTIONS
301	1.03	1.11				INCLUDED IN RANGE
302	1.09	1.09		FINE	1.03-1.13	15
303	1.06	1.06		COARSE	1.01-1.12	15
304	1.07	1.07				
305	1.03	1.03		**AVERAGE**	**(D)**	**NUMBER OF SECTIONS**
306	1.05	1.05				**INCLUDED IN AVERAGE**
307	1.07	1.02		FINE	1.07	15
308	1.08	1.08		COARSE	1.06	15
309	1.06	1.06				
310	1.07					
311		1.01				
312	1.08	1.08				
313	1.08	1.06				
314	1.13					
315	1.05	1.05				
316		1.12				
317	1.07	1.07				

APPENDIX 3D						
STATISTICS OF APPALACHIAN PLATEAU SECTIONS						
SECTION NUMBER	**FINE (D)**	**COARSE (D)**		**RANGE**	**(D)**	**NUMBER OF SECTIONS**
401	1.11	1.10				**INCLUDED IN RANGE**
402	1.09	1.03		FINE	1.09-1.13	3
403	1.13	1.08		COARSE	1.03-1.20	4
404		1.20				
				AVERAGE	**(D)**	**NUMBER OF SECTIONS**
						INCLUDED IN AVERAGE
				FINE	1.11	3
				COARSE	1.10	4